The History and Concept of Computability

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1   Abstract

This lecture will trace the history and development of the concept of computability, its early emergence, its definition under Turing in 1936 and the notions of effective calculability and computable enumerability by other researchers.

Although this definition of Turing machine computable functions is the one which convinced Gödel and is the most widely accepted today, it represents *closed* computability and is not the notion of most interest today in either theoretical research or practical applications. Most of these modern results, theoretical and practical, take place in a more general setting of *open* computing. This concept, appeared almost accidentally at first, and took several decades to develop. It has often been underestimated, or ignored, even by experts, and has never been given the same status as Turing machines.

It encompasses various computable approximations, such as machine learning, and decision making under uncertainty, so prevalent in real world computing, such as the financial markets. It also includes the foundation of most results in theoretical computability theory, whether dealing with pure computability, or applications to other areas of logic and mathematics such as model theory, algebra, number theory, and differential geometry. During the second half of this talk, we develop the properties of open computing, its algorithmic, topological, and definability properties. We present the case that it is the central concept in computability theory.

2   CCA Conference

Robert Soare will deliver this invited lecture at the *Third International Conference on Computability and Complexity in Analysis,* November 1–5, 2006 in Gainshesville, Florida. The conference announcement states,

We hope to gain new insights into computability-theoretic aspects of various computatational questions ....”

This lecture is intended to address such central concepts of computability.