A. Discussion on modeling assumptions

We begin this section with a discussion on our rationale behind the modelling assumptions made in this paper.

- Workers maximize their expected payment: In the literature on game theory, this assumption is a standard, albeit highly debated, assumption. We argue that this assumption is quite reasonable in our setting. In standard labeling tasks in crowdsourcing, workers typically spend only about a few minutes for each task, and participate in hundreds of tasks every week. As a consequence of the law of large numbers, their earning per hour quickly converges to its expected value. Assuming that workers aim to maximize their hourly wages, the expected payment is the correct quantity to consider.

- Cost-of-effort: This choice of not explicitly modelling a "cost-for-effort" of each worker was guided by the principle of Occam's razor. The cost-for-effort is a highly complex quantity and is not very well understood. (For instance, what is the monetary cost for the effort in writing or reading this paper?). Hence, instead, we consider the parameter μ to be a surrogate for the cost-for-effort: the parameter must be scaled in a fashion that ensures a expected fair pay to any worker who does a reasonable job.

- Workers perfectly know their beliefs: We admit this is a mathematical idealization, but is somewhat necessary to enable a principled game-theoretic analysis of the setting, and is quite a standard assumption in the literature.

- Non-negative payments: To the best of our knowledge, all crowdsourcing platforms today (such as Amazon mechanical turk, Clickworker, Mobileworks, etc.) require the payment to be non-negative.

- Rational workers: We do not require workers to be rational; rationality is a standard game theoretic assumption employed to guard against the worst case of workers exploiting the payment mechanism. From a practical standpoint, workers exposed to any mechanism for long enough durations may eventually "rationalize" and identify loopholes (if any) in the mechanism.

B. Simulations for SVM with RBF Kernel

In this section, we plot the results of the simulations for the SVM algorithm with the RBF kernel. To begin, Figure 5 plots the error incurred when the number of workers in the setting with no self correction is varied from 5 to 9, keeping the number of workers in the setting with self correction at 5. Next, Figure 6 compares the error in the two settings when q is fixed at 0.15 for various values of parameter p. Finally, Figure 7 compares the error in the two settings when p is fixed at 0.6 for various values of parameter q. We observe that as in the case of the linear kernel studied earlier, the two-stage setting with self correction offers significant advantages over the single-stage setting with no self correction.

C. Proofs

In this section, we will present the proofs of the various theoretical claims made in the main text. We begin with the claim for the single-stage setting followed by the proofs of the main two-stage setting considered in the paper. Towards the latter, in Section C.2, we introduce some notation and a lemma that will subsequently be used in several other proofs.

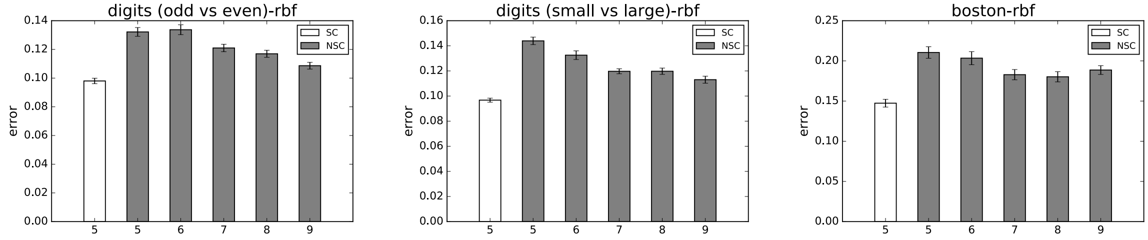

Figure 5: Error incurred by SVM with an RBF kernel under the self-correction (SC) setting with 5 workers, compared to the error incurred under the standard setting with no self correction (NSC) with 5 to 9 workers.

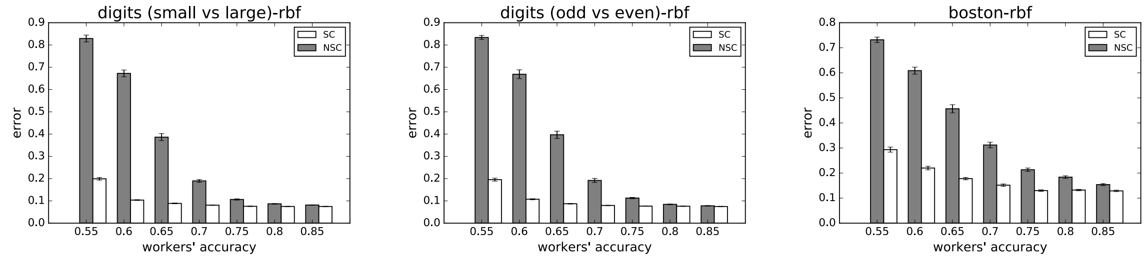

Figure 6: Error incurred by SVM with an RBF kernel for different reliabilities (p) of the worker in the first stage. The no-self-correction (NSC) setting has 7 workers whereas the self-correction (SC) setting has only 5 workers.

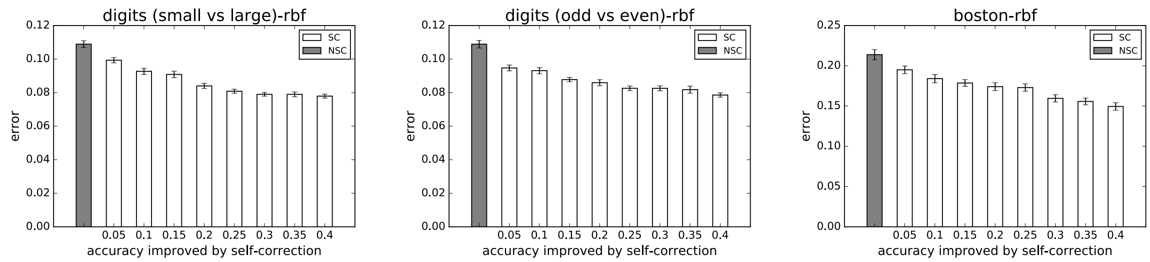

Figure 7: Error incurred by SVM with an RBF kernel for different values of the improvement in accuracy (q) via self-correction. The no-self-correction (NSC) setting has 7 workers whereas the self-correction (SC) setting has only 5 workers.

C.1. Proof of Proposition 1: One stage is easy

The proof is straightforward, but is included for completeness. Let p_A and p_B $(= 1 - p_A)$ be the worker's subjective probabilities of A or B respectively being correct. If the worker selects A then her expected reward is

$$R_A := p_A M_+ + p_B M_- \ .$$

On the other hand, if the worker selects B then her expected reward is

$$R_B := p_B M_+ + p_A M_- \ .$$

Noting that $p_A + p_B = 1$, one can easily verify that

$$M_+ > M_- \Rightarrow R_A \underset{p_A > \frac{1}{2} > p_B}{\overset{p_A < \frac{1}{2} < p_B}{\lesseqgtr}} R_B$$

which implies incentive compatibility.

C.2. Necessary and sufficient condition for incentive compatibility when $N = G = 1$

In this section, we establish a key result on necessary and sufficient conditions for incentive compatibility when $N = G = 1$, which will be useful in subsequent proofs. The reader interested in only the proof of Theorem 1 may directly read that proof in the next subsection without loss in continuity.

Under the special case of $N = G = 1$, any mechanism $f : \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\} \rightarrow [0, \mu]$ can be defined using six values in the interval $[0, \mu]$, namely $M_+ := f(+\mathfrak{M})$, $M_- := f(-\mathfrak{M})$, $R_+ := f(+\mathfrak{R})$, $R_- := f(-\mathfrak{R})$, $C_+ := f(+\mathfrak{C})$ and $C_- := f(-\mathfrak{C})$.

We will also use the following two functions $R_R, R_C : [0, 1] \rightarrow [0, \mu]$:

$$R_R(p') := p' R_+ + (1 - p') R_-, \tag{2a}$$

$$R_C(p') := (1 - p') C_+ + p' C_- \ . \tag{2b}$$

In words, $R_R(p')$ and $R_C(p')$ represent the expected reward of a worker (from her point of view) who has a belief of p' in the option she chose in the first stage and who either retains her answer or copies the reference answer respectively. In this section, since we consider only one question, we will drop the subscripts "i" in the notation of the worker's beliefs.

The following lemma establishes necessary and sufficient conditions for incentive compatibility.

Lemma 1. *When $N = G = 1$, a necessary and sufficient condition for a mechanism to be incentive compatible is that it satisfies the following conditions:*

$$(1-T)R_+ + TR_- = TC_+ + (1-T)C_-, \tag{3a}$$

$$\max\left\{C_+, \frac{C_+ + C_-}{2} + 2\xi\frac{C_+ - C_-}{2}, \frac{R_+ + R_-}{2} - 2\xi\frac{R_+ - R_-}{2}\right\} \leq \frac{M_+ + M_-}{2} + 2\xi\frac{M_+ - M_-}{2}, \tag{3b}$$

$$\frac{M_+ + M_-}{2} - 2\xi\frac{M_+ - M_-}{2} \leq \min\left\{C_+, TC_+ + (1-T)C_-, \max\{\frac{C_+ + C_-}{2} + 2\xi\frac{C_+ - C_-}{2}, \frac{R_+ + R_-}{2} - 2\xi\frac{R_+ - R_-}{2}\}\right\}, \tag{3c}$$

$$M_+ > M_-, \qquad R_+ > C_-, \qquad C_+ > R_-, \qquad R_+ > M_-. \tag{3d}$$

The remainder of this subsection is devoted to the proof of this lemma.

Proof of Lemma 1 We will prove this lemma by first identifying the basic conditions necessary and sufficient for incentive compatibility, and then showing the equivalence of the conditions to those stated in the lemma.

Recall the conditions for incentive compatibility in the second stage (Section 2.3). One can verify that equivalently, necessary and sufficient conditions for incentive compatibility in the second stage are $R_R(1-T) = R_C(1-T)$ and $R_+ > C_-$, $C_+ > R_-$. The first condition is identical to (3a).

For the first stage, by definition, a necessary and sufficient condition for incentive compatibility is

$$\begin{aligned} &q_A(p_A M_+ + p_B M_-) + q_B \max\{R_R(p'_{A|B}), R_C(p'_{A|B})\} \\ &\underset{p_A > \frac{1}{2}+\xi}{\overset{p_A < \frac{1}{2}-\xi}{\lessgtr}} q_B(p_B M_+ + p_A M_-) + q_A \max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}, \end{aligned} \tag{4}$$

for all $p_A \in [0,1], p'_{A|B} \in [0, p_A], p'_{B|A} \in [0, p_A], p'_{B|A} \in [0, p_B]$, and $q_A \in [0,1]$.

Setting $p'_{B|A} = p'_{A|B} = 0$ and $q_A = q_B = \frac{1}{2}$ in (4) results in the necessity of the condition $M_+ > M_-$. Let us now investigate the conditions (3b) and (3c).

Consider the case of $p_A < \frac{1}{2} - \xi$. Here, the worst case is when the left hand side of (4) is maximized and the right hand side is minimized. Satisfying (4) when $p_A < \frac{1}{2} - \xi$ is thus equivalent to satisfying the inequality

$$\begin{aligned} &q_A(p_A M_+ + p_B M_-) + q_B \max_{p'_{A|B} \in [0,p_A]} \max\{R_R(p'_{A|B}), R_C(p'_{A|B})\} \\ &< q_B(p_B M_+ + p_A M_-) + q_A \min_{p'_{B|A} \in [0,p_B]} \max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}. \end{aligned} \tag{5}$$

Recall that $q_A = 1 - q_B$. Observe that the inequality (5) is linear in q_A. As a result, a necessary and sufficient for (5) to be satisfied for all values of $q_A \in [0,1]$ is that the inequality (5) is satisfied for the two extreme values of q_A, namely $q_A \in \{0,1\}$. Setting $q_A = 0$ in (5) gives

$$\max_{p'_{A|B} \in [0,p_A]} \max\{R_R(p'_{A|B}), R_C(p'_{A|B})\} < (p_B M_+ + p_A M_-). \tag{6}$$

The 'maximum' term in the left hand side of (6) is a maximum over two linear functions, and hence the term is maximized when $p'_{A|B}$ is either 0 or p_A. Thus (6) reduces to

$$\max\{R_-, C_+, R_R(p_A), R_C(p_A)\} < (p_B M_+ + p_A M_-),$$

for all $p_A \in [0, \frac{1}{2} - \xi)$. Using the condition $C_+ > R_-$ from (3d), we obtain the equivalent condition

$$\max\{C_+ - (p_B M_+ + p_A M_-), R_R(p_A) - (p_B M_+ + p_A M_-), R_C(p_A) - (p_B M_+ + p_A M_-)\} < 0. \tag{7}$$

Each of the three expressions in the maximum on the left hand side of (7) are linear expressions in terms of the variable p_A. Consequently, the maximum is attained at one of the end-points of the permitted values of p_A, that is, when $p_A = 0$ or when p_A approaches $\frac{1}{2} - \xi$. Substituting these two values of p_A into (7) yields the necessary and sufficient condition of (3b), for the setting of $p_A < \frac{1}{2} - \xi$ and $q_A = 0$.

Next we move to the case of $q_A = 1$. Setting $q_A = 1$ in (5) gives

$$(p_A M_+ + p_B M_-) < \min_{p'_{B|A} \in [0, p_B]} \max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}. \tag{8}$$

The term "$\max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}$" in the right hand side of (8) is a maximum over two linear functions, and hence the term is necessarily minimized in one of the following three cases: (i) At $R_R(p'_{B|A}) = R_C(p'_{B|A})$ if one of the two functions $R_R(p'_{B|A})$ and $R_C(p'_{B|A})$ is increasing and one decreasing in $p'_{B|A}$. As a consequence of (3a), the two functions are equal when $p'_{B|A} = 1 - T$. Note that this value of $p'_{B|A}$ is a valid value because $1 - T \leq \frac{1}{2} \leq p_B$. (ii) At $p'_{B|A} = 0$, which is a minimizer when both functions increase with an increase $p'_{B|A}$. (iii) At $p'_{B|A} = p_B$, which is a minimizer when both functions decrease with an increase $p'_{B|A}$. Putting the three cases together, we get the equivalent condition

$$(1 - p_B)M_+ + p_B M_- < \min\{C_+, TC_+ + (1-T)C_-, \max\{R_R(p_B), R_C(p_B)\}\}, \tag{9}$$

for all $p_B \in [0, \frac{1}{2} - \xi)$. One can verify that due to linearity (in p_B) of the various constituents of (9), it is necessary and sufficient that the inequality (9) be satisfied for the extreme values of p_B. Setting $p_B = 1$ and $p_B = \frac{1}{2} + \xi$ and performing some algebraic simplifications yields the condition (3c).

The case of $p_B < \frac{1}{2} - \xi$ gives the same result by symmetry. This completes the proof of the necessity and sufficiency of (3) for incentive compatibility.

C.3. Proof of Theorem 1: Impossibility

We first prove the claimed impossibility result for the case of a single question $N = G = 1$. The proof for the case of $N = G = 1$ proceeds via a contradiction-based argument, and uses the notation of Section C.2.[6] Suppose there is an incentive compatible mechanism, i.e., there exist values of $M_+, M_-, R_+, R_-, C_+, C_-$ that ensure that in both stages the worker selects the answer she thinks is most likely to be correct.

Incentive compatibility then necessitates:

- Second stage:
  - if worker answered A in the first stage and reference answer was B:

$$R_R(p'_{A|B}) \underset{p'_{A|B} > 1-T}{\overset{p'_{A|B} < 1-T}{\lesseqgtr}} R_C(p'_{A|B}), \tag{10}$$

  - if worker answered B in the first stage and reference answer was A:

$$R_R(p'_{B|A}) \underset{p'_{B|A} > 1-T}{\overset{p'_{B|A} < 1-T}{\lesseqgtr}} R_C(p'_{B|A}). \tag{11}$$

- First stage:

$$q_A(p_A M_+ + p_B M_-) + q_B \max\{R_R(p'_{A|B}), R_C(p'_{A|B})\} \underset{p_A > \frac{1}{2} > p_B}{\overset{p_A < \frac{1}{2} < p_B}{\lesseqgtr}} q_B(p_B M_+ + p_A M_-) + q_A \max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}. \tag{12}$$

[6]While one could use Lemma 1 to prove this result, we opt for a different proof here for its significantly greater simplicity.

We now show that the requirements (10), (11) and (12) cannot be met simultaneously. To this end, consider some value $p' \in [0, \frac{1}{2}]$, and consider a worker who has subjective probabilities $p_A = p_B = \frac{1}{2}$, $p'_{A|B} = p'_{B|A} = p' \leq \frac{1}{2}$, and $q_A \neq q_B$. Observe that both the left and right hand sides of (12) are continuous in (p_A, p_B). As a result, when $p_A = p_B = \frac{1}{2}$ we must have

$$\begin{aligned} &q_A(\frac{1}{2}M_+ + \frac{1}{2}M_-) + q_B \max\{R_R(p'_{A|B}), R_C(p'_{A|B})\} \\ &= q_B(\frac{1}{2}M_+ + \frac{1}{2}M_-) + q_A \max\{R_R(p'_{B|A}), R_C(p'_{B|A})\}. \end{aligned}$$

Some simple algebraic manipulations yield

$$\frac{M_+ + M_-}{2} = \max\{R_R(p'), R_C(p')\}, \tag{13}$$

for every $p' \leq \frac{1}{2}$. In the two sets of inequalities (10) and (11), the left hand sides are greater than the right hand sides for certain values of $p' \leq \frac{1}{2}$, and vice versa for certain other values of $p' \leq \frac{1}{2}$, whenever $T > \frac{1}{2}$. It follows that the term $\max\{R_R(p'), R_C(p')\}$ in the right hand side of (13) must depend on the value of p' and cannot be a constant. On the other hand, the left hand side of (13) is a constant, independent of p'. This argument thus yields a contradiction.

Given that the worker cannot be incentivized for even one question, the impossibility easily extends to the more general case of $N \geq G \geq 1$ as follows. Assume that for questions $2, \ldots, N$, the worker is sure that the answer is option A in both stages, is sure that the reference answer will be option A, and the reference answer as well as the correct answer actually turn out to equal option A. In this setting, the incentivization requirements reduce to incentivizing the worker for only the first question, which is shown to be impossible in the proof for the $N = G = 1$ setting below.

C.4. Proof of Theorem 2: Many mechanisms for every slack

We begin with the case of $N = G = 1$ which will convey many of the key ideas of the proof. We will adopt the notation introduced in Section C.2. Let $M_+ = 1, M_- = 0, R_+ = 1, R_- = 0, C_+ = (1-T), C_- = (1-T)$. It is easy to verify that this choice satisfies the conditions (3a) and (3d). If these payments satisfy the inequalities (3b) and (3c), then we are done. If not then the values will result in the left hand side being greater than the right hand side in (3b) and/or (3c). In that case, compute the difference between the left and right hand sides of (3b) and (3c), and let $\zeta > 0$ denote the larger of the two values. Perform the following modifications to the values: $M_+ \to M_+ + \frac{\zeta+1}{\xi}$, $M_- \to M_-$, $R_+ \to R_+ + \frac{\zeta+1}{2\xi}$, $R_- \to R_- + \frac{\zeta+1}{2\xi}$, $C_+ \to C_+ + \frac{\zeta+1}{2\xi}$, and $C_- \to C_- + \frac{\zeta+1}{2\xi}$. At this point, we would like to remind the reader that $\zeta > 0$ and $\xi > 0$.

One can verify that with the changes described above, the payment values continue to satisfy the conditions (3a) and (3d). However, importantly, with these changes, the left hand side of (3b) increases by at most $\frac{\zeta+1}{2\xi}$ while the right hand side increases by $(1+2\xi)\frac{\zeta+1}{2\xi}$, and the left hand side of (3b) increases by $(1-2\xi)\frac{\zeta+1}{2\xi}$ while its right hand side increases by $\frac{\zeta+1}{2\xi}$. It follows that in both inequalities, the difference between the right and left hand sides increases by at least $(\zeta+1)$. Thus with the updated values, both (3b) and (3c) are satisfied. Finally, scaling all payments by $\frac{\mu}{M_+}$ also ensures that the mechanism abides by the constraint of the maximum allowable payment. We have thus proved the existence of an incentive-compatible mechanism when $\xi > 0$, for the case of $N = G = 1$.

For the more general case of $N \geq G \geq 1$, consider a mechanism that first considers each gold standard question separately and allots a score equaling the payment that would have been made in the case of $N = G = 1$. The net payment across all questions is the sum of the scores across all questions (normalized by a positive factor to satisfy the budget constraint of μ). From the worker's point of view, due to linearity of expectation, the expected payment for any choice of answers is the sum of the expected scores for the N individual questions (normalized by a positive constant factor). Incentive compatibility of the individual scores for $N = G = 1$ implies incentive compatibility for the general mechanism as well.

Observe that in the proof above, we started out with one particular choice of the parameters M_+, M_-, R_+, R_-, C_+, C_- that satisfied (3a) and (3d). There are however infinitely many choices of these parameters that satisfy these two conditions. The rest of the proof for $G = 1$ above demonstrated a procedure to construct an incentive-compatible mechanism starting from any such choice. It is not hard to see that the set of resulting mechanisms also form an infinite set. When $G > 1$, one can choose separate mechanisms for each individual question and combine them in one of an exponentially large number of ways, e.g., multiplying or adding any of the mechanisms for the individual questions. The number of degrees of freedom thus grows exponentially in G.

C.5. Proof of Theorem 3: Minimum slack needed

We begin with the case of $N = G = 1$ which will convey many of the key ideas of the proof. We will adopt the notation introduced in Section C.2 for this case.

It is straightforward to see that when $N = G = 1$, a necessary and sufficient condition to satisfy the no-free-lunch axiom is that $M_- = R_- = C_- = 0$. Substituting these conditions in Lemma 1 gives that a necessary and sufficient condition under any ξ and T for the existence of an incentive-compatible mechanism satisfying the no-free-lunch axiom is

$$\frac{\frac{1}{2} - \xi}{T} \leq \frac{C_+}{M_+} \leq \frac{\frac{1}{2} + \xi}{T} \min\left\{\frac{1 - T}{\frac{1}{2} - \xi}, T\right\} \tag{14a}$$

$$(1 - T)R_+ = TC_+ > 0. \tag{14b}$$

Observe the following four properties of (14a): (i) the leftmost side strictly decreases with an increase in ξ while its rightmost side increases strictly, (ii) when $\xi = 0$, the leftmost side is strictly greater than its rightmost side (using the fact that $T < 1$), (iii) when $\xi = \frac{1}{2}$, the leftmost side is zero whereas the rightmost side is one, and (iv) both the leftmost and rightmost sides are continuous in ξ. It follows that the leftmost and rightmost sides of (14a) meet each other at exactly one point in $\xi \in (0, \frac{1}{2})$. Solving (14a) for ξ, with the inequalities are replaced by equalities, gives precisely the value denoted by $\xi_{\min}$ in the statement of the theorem. For any $\xi < \xi_{\min}$, the aforementioned arguments imply a violation of (14a).

Let us now consider the more general case of $N \geq G \geq 1$. Suppose there exists some value $\xi < \xi_{\min}$ for which there exists an incentive compatible mechanism satisfying the no-free-lunch axiom. Then we have

$$\frac{1}{\frac{1}{2} - \xi} = \frac{1}{\frac{1}{2} + \xi} \max\left\{\frac{\frac{1}{2} - \xi}{1 - T}, \frac{1}{T}\right\} - \delta_{\xi,T}, \tag{15}$$

for some value $\delta_{\xi,T} > 0$ that depends on the values of ξ and T. We will now call upon the proof of Theorem 5 to complete our proof. The proof of Theorem 5 shows that under the no-free-lunch axiom, there is only one mechanism that can be incentive compatible when $\xi = \xi_{\min}$. In the proof of Theorem 5, the steps till Equation (16) are applicable to all values of $\xi \in (0, \frac{1}{2})$; ξ is set as $\xi_{\min}$ in (16) to obtain (17) and in subsequent steps. If $\xi_{\min}$ is replaced by ξ, then the inequality (20) becomes

$$\left(\frac{1}{2} + \xi\right) \bar{h}(-\mathfrak{M}) + \frac{\frac{1}{2} - \xi}{\frac{1}{2} + \xi} T\bar{h}(+\mathfrak{C})\delta_{\xi,T} \leq \frac{\left(\frac{1}{2} - \xi\right)^2}{\frac{1}{2} + \xi} \bar{h}(-\mathfrak{M}),$$

where $\bar{h}(-\mathfrak{M}) \geq 0$, $\bar{h}(+\mathfrak{C}) > 0$. One can see that when $\xi > 0$, a necessary condition for this inequality to be satisfied (and consequently for any mechanism to be incentive compatible) is to have $\delta_{\xi,T} = 0$. This assignment, in turn, necessitates $\xi = \xi_{\min}$ for existence of any incentive compatible mechanism, as claimed.

C.6. Proof of Theorem 4: The algorithm works

First consider the case of $N = G = 1$. One can verify that when $\xi = \xi_{\min}$, the proposed payment mechanism satisfies the necessary and sufficient conditions (3) derived earlier in Lemma 1.

For the case of $N \geq G \geq 1$, observe that the mechanism assigns a non-negative value to the worker for each question in the gold standard, and the final payment is the product of these values (scaled by a positive constant). We recall our assumption that the worker's beliefs are independent across questions. Consequently, in either stage, the net expected payment from the worker's point of view equals the product of the expected values for each question (where the value is 1 if the question is not in the gold standard). Since for every individual question, the expectation is maximized when the worker answers as desired, the overall expected payment is also maximized when the worker answers as desired. The mechanism is thus incentive compatible.

Finally, one can verify that the payment is always non-negative, and the maximum payment equals μ.

C.7. Proof of Theorem 5: One and only mechanism

Let us first consider the much simpler case of $N = G = 1$. Recall the necessary and sufficient conditions (3) for incentive compatibility with the no free lunch condition. When $\xi = \xi_{\min}$, the two inequalities in (14a) get tightly sandwiched and transform into equalities. Thus, the parameters M_+, R_+ and C_+ now have a unique relation between them. Moreover, $M_- = R_- = C_- = 0$ are also fixed. Setting $\max\{M_+, R_+, C_+\} = \mu$ now fixes the entire mechanism to be identical to Algorithm 1.

We now move on to the case of general values of the parameters (N, G). We begin with two lemmas that derive properties that any mechanism must necessarily satisfy. The proofs of these two lemmas are provided at the end of this section. The first of the two lemmas applies to any incentive compatible mechanism, that may or may not satisfy no free lunch.

Lemma 2. *For any $i \in [G]$, any $(y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_G) \in \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^{G-1}$, any incentive compatible mechanism must satisfy*

$$\begin{aligned}(1-T)f(y_1, \ldots, y_{i-1}, +\mathfrak{R}, y_{i+1}, \ldots, y_G) + Tf(y_1, \ldots, y_{i-1}, -\mathfrak{R}, y_{i+1}, \ldots, y_G)\\ = Tf(y_1, \ldots, y_{i-1}, +\mathfrak{C}, y_{i+1}, \ldots, y_G) + (1-T)f(y_1, \ldots, y_{i-1}, -\mathfrak{C}, y_{i+1}, \ldots, y_G).\end{aligned}$$

The proof of Theorem 5 inducts on the number of entries in $\mathbf{y}$ that take values in the set $\{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}$. The hypothesis of this induction is that the payment mechanism must be of the form given in Algorithm 1 up to a constant positive scaling. The base case of $\mathbf{y} \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{R}, +\mathfrak{C}\}^G$ is handled in Lemma 3 below.

Lemma 3. *Any incentive-compatible mechanism satisfying the no-free-lunch axiom must satisfy $f(\mathbf{y}) = 0 \ \forall \ \mathbf{y} \in \{-\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{R}, +\mathfrak{C}\}^G$.*

From Lemmas 2 and 3, we obtain the base case of the induction that the mechanism must be identical to that of Algorithm 1 whenever $\mathbf{y} \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{R}, +\mathfrak{C}\}^G$.

Moving on, let us now suppose that the induction hypothesis is true whenever $\mathbf{y} \in \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{M}, +\mathfrak{R}, +\mathfrak{C}\}^G$ and $\sum_{i=1}^G \mathbf{1}\{y_i \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}\} \geq G - \gamma + 1$, for some $\gamma \in [G]$. We now prove that the induction hypothesis remains true whenever $\mathbf{y} \in \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{M}, +\mathfrak{R}, +\mathfrak{C}\}^G$ and $\sum_{i=1}^G \mathbf{1}\{y_i \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}\} = G - \gamma$.

Suppose that without loss of generality that $y_1, \ldots, y_{\gamma-1} \in i \in \{+\mathfrak{M}, -\mathfrak{M}\}$ and $y_{\gamma+1}, \ldots, y_G \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}$. In the total set of N questions, suppose that for every $i \leq \gamma - 1$, we have $q_{A,i} = 1$, $p_{A,i} > \frac{1}{2} + \xi$, and for every $i \geq \gamma + 1$, we have $q_{A,i} = 0$, $p_{A,i} > \max\{\frac{1}{2} + \xi, T\}$. Suppose that for all questions $[N] \backslash \{\gamma\}$, the worker decides to act precisely as what the mechanism wishes her to do. Thus in the first stage, she will select option A for all questions $[N] \backslash \{\gamma\}$. Furthermore, the worker believes that questions $1, \ldots, \gamma - 1$ will surely match, whereas questions $\gamma + 1, \ldots, G$ will surely mismatch and go into the second stage.

Let $h : \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\} \rightarrow [0, \mu]$ be a function defined as follows: $h(y_\gamma)$ is the expected payment, from the point of view of the worker, conditioned on the γ^{th} question evaluating to y_γ. (Note that since $q_{A,i} \in \{0, 1\}$ for every $i \neq \gamma$, and since the evaluation of question γ is fixed at y_γ, the expected pay is identical in both stages.) The expectation is over the randomness in the choice of the gold standard questions as well as over the worker's uncertainty about the correctness of her answers to the remaining $N - 1$ questions. One can see that for any value of y_γ, the function $h(y_\gamma)$ is composed of a convex combination of two parts: the first part is for the case when the γ^{th} question is in the gold standard and the second part is when the γ^{th} question is not in the gold standard. Consequently, the first part depends on y_γ and the second part is independent of it. Letting $\bar{h}$ denote the first part, we can write $h(y_\gamma) = \theta\bar{h}(y_\gamma) + (1-\theta)c$ for some constants $c \geq 0$ and $\theta \in (0, 1)$.

The function $\bar{h}$ is a convex combination of the function f evaluated at various points. In particular, when $y_\gamma \in \{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}$, each component of this convex combination is the function f evaluated at a vector with at least $(G - \gamma + 1)$ of its entries taking values in the set $\{+\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}$. Hence applying Lemma 2 we get that $(1-T)\bar{h}(+\mathfrak{R}) + T\bar{h}(-\mathfrak{R}) = T\bar{h}(+\mathfrak{C}) + (1-T)\bar{h}(-\mathfrak{C})$. Furthermore, from our induction hypothesis above, we have $\bar{h}(y_\gamma) = 0$ when $y_\gamma \in \{-\mathfrak{R}, -\mathfrak{C}\}$. Consequently, we also have $\bar{h}(+\mathfrak{R}) = \frac{T}{1-T}\bar{h}(+\mathfrak{C})$.

Let $p_A, p_B = 1 - p_A, q_A, q_B = 1 - q_A, p'_{A|B}, p'_{B|A}$ be the confidences of the worker for question γ. In order to incentivize the worker appropriately for question γ in the first stage, it must be that

$$\begin{aligned} & q_A(p_A\bar{h}(+\mathfrak{M}) + p_B\bar{h}(-\mathfrak{M})) + q_B \max\{p'_{A|B}\bar{h}(+\mathfrak{R}), (1 - p'_{A|B})\bar{h}(+\mathfrak{C})\} \\ & \underset{p_A > \frac{1}{2}+\xi}{\overset{p_A < \frac{1}{2}-\xi}{\lesseqgtr}} q_B(p_B\bar{h}(+\mathfrak{M}) + p_A\bar{h}(-\mathfrak{M})) + \max\{p'_{B|A}\bar{h}(+\mathfrak{R}), (1 - p'_{B|A})\bar{h}(+\mathfrak{C})\}. \end{aligned}$$

Substituting $\bar{h}(+\mathfrak{R}) = \frac{T}{1-T}\bar{h}(+\mathfrak{C})$ we get

$$q_A(p_A\bar{h}(+\mathfrak{M}) + p_B\bar{h}(-\mathfrak{M})) + q_B \max\{p'_{A|B}\frac{T}{1-T}, (1-p'_{A|B})\}\bar{h}(+\mathfrak{C})$$
$$\underset{p_A > \frac{1}{2}+\xi}{\overset{p_A < \frac{1}{2}-\xi}{\lesseqgtr}} q_B(p_B\bar{h}(+\mathfrak{M}) + p_A\bar{h}(-\mathfrak{M})) + q_A \max\{p'_{B|A}\frac{T}{1-T}, (1-p'_{B|A})\}\bar{h}(+\mathfrak{C})\ .$$

Let $p_A = \frac{1}{2} - \xi$. Setting $p'_{B|A} = 1 - T$ and allowing $p'_{A|B}$ to be 0 or p_A gives

$$q_A\left(\frac{1}{2}-\xi\right)\bar{h}(+\mathfrak{M}) + q_A\left(\frac{1}{2}+\xi\right)\bar{h}(-\mathfrak{M}) + q_B T \max\left\{\left(\frac{1}{2}-\xi\right)\frac{1}{1-T}, \frac{1}{T}\right\}\bar{h}(+\mathfrak{C})$$
$$\leq q_B\left(\frac{1}{2}+\xi\right)\bar{h}(+\mathfrak{M}) + q_B\left(\frac{1}{2}-\xi\right)\bar{h}(-\mathfrak{M}) + q_A T\bar{h}(+\mathfrak{C}).$$

From the definition of the minimum slack (Theorem 3), when $\xi = \xi_{\min}$, we have

$$\frac{\frac{1}{2}-\xi}{T} = \frac{\frac{1}{2}+\xi}{T}\min\left\{\frac{1-T}{\frac{1}{2}-\xi}, T\right\}\ , \tag{16}$$

and hence

$$q_A\left(\frac{1}{2}-\xi\right)\bar{h}(+\mathfrak{M}) + q_A\left(\frac{1}{2}+\xi\right)\bar{h}(-\mathfrak{M}) + q_B T\frac{\frac{1}{2}+\xi}{\frac{1}{2}-\xi}\bar{h}(+\mathfrak{C})$$
$$\leq q_B\left(\frac{1}{2}+\xi\right)\bar{h}(+\mathfrak{M}) + q_B\left(\frac{1}{2}-\xi\right)\bar{h}(-\mathfrak{M}) + q_A T\bar{h}(+\mathfrak{C}). \tag{17}$$

Setting $q_A = 1$ gives

$$\left(\frac{1}{2}-\xi\right)\bar{h}(+\mathfrak{M}) + \left(\frac{1}{2}+\xi\right)\bar{h}(-\mathfrak{M}) \leq T\bar{h}(+\mathfrak{C}), \tag{18}$$

and setting $q_A = 0$ gives

$$T\frac{\frac{1}{2}+\xi}{\frac{1}{2}-\xi}\bar{h}(+\mathfrak{C}) \leq \left(\frac{1}{2}+\xi\right)\bar{h}(+\mathfrak{M}) + \left(\frac{1}{2}-\xi\right)\bar{h}(-\mathfrak{M}). \tag{19}$$

Combining the inequalities (18) and (19) yields the bound

$$\left(\frac{1}{2}+\xi\right)\bar{h}(-\mathfrak{M}) \leq \frac{\left(\frac{1}{2}-\xi\right)^2}{\frac{1}{2}+\xi}\bar{h}(-\mathfrak{M}). \tag{20}$$

Since $\xi \in (0, \frac{1}{2})$, the inequality (20) can be satisfied only if $\bar{h}(-\mathfrak{M}) = 0$. The function $\bar{h}(-\mathfrak{M})$ is a convex combination of various evaluations of the non-negative function f including $f(y_1, \ldots, y_G)$. It follows that these evaluations of f must also be zero. We have thus proved that

$$f(\mathbf{y}) = 0 \quad \forall\, \mathbf{y} \in \{+\mathfrak{M}, -\mathfrak{M}, +\mathfrak{R}, -\mathfrak{R}, +\mathfrak{C}, -\mathfrak{C}\}^G \backslash \{+\mathfrak{M}, +\mathfrak{R}, +\mathfrak{C}\}^G. \tag{21}$$

Continuing on, substituting the result of (21) in (18) and (19) yields the relation

$$T\bar{h}(+\mathfrak{C}) = \left(\frac{1}{2}-\xi\right)\bar{h}(+\mathfrak{M}). \tag{22}$$

We now convert this relation of the function $\bar{h}$ to an analogous relation of the function f. Suppose that for every question $i \in \{G+1, \ldots, N\}$, the worker has beliefs $p_{A,i} = 1,\ p'_{A|B,i} = 1,\ q_{A,i} = 0$, and that every question in this set actually results in a mismatch. Recall that the function $\bar{h}$ is a convex combination of the function f evaluated at various points corresponding to the various choices of the G gold standard questions out of the N total questions, where the choice necessarily includes question 1. Applying this observation to the relation (22) yields

$$\sum_{\substack{j\in\{0,\ldots,G-1\}\\ i_1,\ldots,i_j\subseteq\{2,\ldots,G\}}} \alpha_{i_1,\ldots,i_j}\Big\{Tf(+\mathfrak{C}, y_{i_1}, \ldots, y_{i_j}, +\mathfrak{R}, \ldots, +\mathfrak{R}) - (\frac{1}{2}-\xi)f(+\mathfrak{M}, y_{i_1}, \ldots, y_{i_j}, +\mathfrak{R}, \ldots, +\mathfrak{R})\Big\},$$

where $\{\alpha_{i_1,\ldots,i_j}\}$ are all positive constants. An inductive argument on the values of $(y_2,\ldots,y_G)$, starting with $y_2=\ldots=y_G=+\mathfrak{R}$ as the base case and further inducting on the number of values in $y_2,\ldots,y_G$ equalling $+\mathfrak{R}$ yields the result

$$Tf(+\mathfrak{C},y_2,\ldots,y_G)=\left(\frac{1}{2}-\xi\right)f(+\mathfrak{M},y_2,\ldots,y_G), \tag{23}$$

for every value of $y_2,\ldots,y_G$. Calling upon Lemma 2 and using (21) also yields

$$Tf(+\mathfrak{C},y_2,\ldots,y_G)=(1-T)f(+\mathfrak{R},y_2,\ldots,y_G). \tag{24}$$

From the relations (21), (23) and (24) and using the fact that all arguments above apply to any permutation of the G gold standard questions yield the claimed result that f must be identical to the mechanism of Algorithm 1.

The only remaining detail is to prove Lemma 2 and Lemma 3 which we do below.

Proof of Lemma 2 We begin by introducing some additional notation that will aid in subsequent discussion. Define a function $g:\{+\mathfrak{M},-\mathfrak{M},+\mathfrak{R},-\mathfrak{R},+\mathfrak{C},-\mathfrak{C}\}^N\to[0,\mu]$ as the expected payment (across the randomness in the choice of the gold standard questions) given the evaluations to all the N questions, that is,

$$g(y_1,\ldots,y_N)=\frac{1}{\binom{N}{G}}\sum_{(i_1,\ldots,i_G)\subseteq\{1,\ldots,N\}}f(y_{i_1},\ldots,y_{i_G}). \tag{25}$$

We first show that the function g must satisfy the relation

$$(1-T)g(y_1,\ldots,y_{N-1},+\mathfrak{R})+Tg(y_1,\ldots,y_{N-1},-\mathfrak{R})=Tg(y_1,\ldots,y_{N-1},+\mathfrak{C})+(1-T)g(y_1,\ldots,y_{N-1},-\mathfrak{C}), \tag{26}$$

for every value of $(y_1,\ldots,y_{N-1})$. To this end, suppose the worker is presently in the second stage. Suppose that the worker's beliefs regarding the various questions are unaffected by the results of matching or mismatching at the end of the first stage. Letting $\mathcal{S}:=\{i\in[N-1]\mid y_i\in\{+\mathfrak{R},-\mathfrak{R},+\mathfrak{C},-\mathfrak{C}\}$, suppose that questions $\mathcal{S}\cup\{N\}$ make it to the second stage. For every $i\in[N]$, let p'_i be the confidence of the worker for the answer that she marked under event y_i. For every $i\in[N-1]$, let $r_i=p'_i$ if $y_i<0$ and $r_i=(1-p'_i)$ if $y_i>0$.[7] Let $E=[\epsilon_1\ \cdots\ \epsilon_{N-1}]\in\{-1,1\}^{N-1}$.

Since the mechanism is incentive compatible, it must be able to appropriately incentivize the worker for the N^{th} question. This condition necessitates

$$\begin{aligned}
&p'\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1y_1,\ldots,-\epsilon_{N-1}y_{N-1},+\mathfrak{R})\prod_{j\in[N-1]}r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&+(1-p')\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1y_1,\ldots,-\epsilon_{N-1}y_{N-1},-\mathfrak{R})\prod_{j\in[N-1]}r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&\underset{p'>1-T}{\overset{p'<1-T}{\lessgtr}}(1-p')\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1y_1,\ldots,-\epsilon_{N-1}y_{N-1},+\mathfrak{C})\prod_{j\in[N-1]}r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&+p'\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1y_1,\ldots,-\epsilon_{N-1}y_{N-1},-\mathfrak{C})\prod_{j\in[N]\setminus\{\gamma\}}r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right).
\end{aligned} \tag{27}$$

The left hand side of (27) is the expected payment if the worker chooses to retain her answer for the N^{th} question, while the right hand side is the expected payment if she chooses to copy the reference answer. Now, note that for any real valued variable q, and for any constants a, b and c,

$$ay\underset{q>c}{\overset{q<c}{\lessgtr}}b\quad\Rightarrow\quad a>0,\ c=\frac{b}{a},\ b>0\ .$$

[7]For ease of exposition, we consider $\{+\mathfrak{M},+\mathfrak{R},+\mathfrak{C}\}$ as "positive" values, and $\{-\mathfrak{M},-\mathfrak{R},-\mathfrak{C}\}$ as the corresponding "negative" values with inverted signs.

Applying this fact and making some simple algebraic manipulations gives

$$\begin{aligned}&(1-T)\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1 y_1,\ldots,-\epsilon_{N-1}y_{N-1},+\mathfrak{R})\prod_{j\in[N-1]} r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&+T\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1 y_1,\ldots,-\epsilon_{N-1}y_{N-1},-\mathfrak{R})\prod_{j\in[N-1]} r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&-T\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1 y_1,\ldots,-\epsilon_{N-1}y_{N-1},+\mathfrak{C})\prod_{j\in[N-1]} r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)\\
&-(1-T)\sum_{E\in\{-1,1\}^{N-1}}\left(g(-\epsilon_1 y_1,\ldots,-\epsilon_{N-1}y_{N-1},-\mathfrak{C})\prod_{j\in[N-1]} r_j^{\frac{1+\epsilon_j}{2}}(1-r_j)^{\frac{1-\epsilon_j}{2}}\right)=0.\end{aligned}\tag{28}$$

The left hand side of this equation is a polynomial in $\{r_1,\ldots,r_{N-1}\}$ which evaluates to zero for a solid $(N-1)$-dimensional box of values of $\{r_1,\ldots,r_{N-1}\}$. It follows that the coefficients of all monomials in this polynomial must be zero, and in particular, the constant term must be zero. The constant term appears when $\epsilon_j=-1\;\forall\;j$ in the summations. This argument thus yields the relation

$$\begin{aligned}(1-T)g(y_1,\ldots,y_{N-1},+\mathfrak{R})&+Tg(y_1,\ldots,y_{N-1},-\mathfrak{R})\\ &=Tg(y_1,\ldots,y_{N-1},+\mathfrak{C})+(1-T)g(y_1,\ldots,y_{N-1},-\mathfrak{C}),\end{aligned}$$

as claimed. Furthermore, since the arguments above are invariant to any permutation of the questions, we get that for any $i\in[N]$, any $(y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_N)\in\{+\mathfrak{M},-\mathfrak{M},+\mathfrak{R},-\mathfrak{R},+\mathfrak{C},-\mathfrak{C}\}^{N-1}$, any incentive compatible mechanism must satisfy

$$\begin{aligned}(1-T)g(y_1,\ldots,y_{i-1},+\mathfrak{R},y_{i+1},\ldots,y_N)&+Tg(y_1,\ldots,y_{i-1},-\mathfrak{R},y_{i+1},\ldots,y_N)\\ &=Tg(y_1,\ldots,y_{i-1},+\mathfrak{C},y_{i+1},\ldots,y_N)+(1-T)g(y_1,\ldots,y_{i-1},-\mathfrak{C},y_{i+1},\ldots,y_N).\end{aligned}\tag{29}$$

It remains to convert the result of Equation (29) to an equivalent condition on the function f as in the statement of the lemma. To this end, suppose that $y_{G+1}=\ldots=y_N=+\mathfrak{R}$. Also suppose without loss of generality that $i=1$. Then expanding the function g in (29) in terms of its constituent components f, we obtain the relation

$$\begin{aligned}\sum_{\substack{j\in\{0,\ldots,G-1\}\\ i_1,\ldots,i_j\subseteq\{2,\ldots,G\}}}\Big\{&\alpha_{i_1,\ldots,i_j}\big((1-T)f(+\mathfrak{R},y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})+Tf(-\mathfrak{R},y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})\\
&\qquad-Tf(+\mathfrak{C},y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})-(1-T)f(-\mathfrak{C},y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})\big)\\
&+\alpha'_{i_1,\ldots,i_j}\big((1-T)f(y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})+Tf(y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})\\
&\qquad-Tf(y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})-(1-T)f(y_{i_1},\ldots,y_{i_j},+\mathfrak{R},\ldots,+\mathfrak{R})\big)\Big\}=0,\end{aligned}\tag{30}$$

where $\left\{\alpha_{i_1,\ldots,i_j},\alpha'_{i_1,\ldots,i_j}\right\}$ are all positive constants. We complete the proof with inductive argument on the values of $(y_2,\ldots,y_G)$. We begin by considering the base case $y_2=\ldots=y_G=+\mathfrak{R}$, for which we obtain the result

$$(1-T)f(+\mathfrak{R},+\mathfrak{R},\ldots,+\mathfrak{R})+Tf(-\mathfrak{R},+\mathfrak{R},\ldots,+\mathfrak{R})=Tf(+\mathfrak{C},+\mathfrak{R},\ldots,+\mathfrak{R})+(1-T)f(-\mathfrak{C},+\mathfrak{R},\ldots,+\mathfrak{R}).$$

from (30). We further induct on the number of values in $y_2,\ldots,y_G$ that equal $+\mathfrak{R}$ in (30), and this inductive argument thus shows that

$$(1-T)f(+\mathfrak{R},y_2,\ldots,y_G)+Tf(-\mathfrak{R},y_2,\ldots,y_G)=Tf(+\mathfrak{C},y_2,\ldots,y_G)+(1-T)f(-\mathfrak{C},y_2,\ldots,y_G),$$

for all possible values of $y_2,\ldots,y_G$. Finally, all arguments above are invariant to any permutation of the questions, and consequently we get the claimed result.

Proof of Lemma 3 We will induct on the number of entries in $\mathbf{y}$ whose values equal either $-\mathfrak{M}$ or $-\mathfrak{R}$ or $-\mathfrak{C}$ or $+\mathfrak{C}$; let us use the notation γ to denote the number of such entries. When $\gamma = G$, the no-free-lunch axiom implies $f(\mathbf{y}) = 0$, where we have used the assumption that $\mathbf{y} \notin \{+\mathfrak{C}, +\mathfrak{R}\}^G$ when applying the no-free-lunch axiom. The statement of the lemma is thus satisfied in this case.

Now suppose that $f(\mathbf{y}) = 0$ whenever $\gamma \geq \gamma_0 + 1$ for some integer $\gamma_0 > 0$. Consider any evaluation $\mathbf{y}$ such that $y_1, \ldots, y_{\gamma_0} \in \{-\mathfrak{M}, -\mathfrak{R}, -\mathfrak{C}, +\mathfrak{C}\}$. Then from the induction hypothesis stated above, we will have $f(\mathbf{y}) = 0$ if additionally we had $y_{\gamma_0+1} \in \{-\mathfrak{M}, -\mathfrak{R}, -\mathfrak{C}, +\mathfrak{C}\}$. Applying Lemma 2 with $i = \gamma_0 + 1$ gives $f(\mathbf{y}) = 0$ when $y_{\gamma_0+1} = +\mathfrak{R}$. This inductive argument completes the proof of the lemma.

C.8. Proof of Theorem 6: No-free-lunch cannot be stronger

Suppose that for every question, the worker has $p_A = \frac{3}{4} = 1 - p_B$, and further suppose that as desired, the worker selects option A for every question in the first stage. Suppose that there is a mismatch for every question, and hence all the questions go to the second stage. Now suppose that in the second stage, the worker has an updated belief $p'_{A|B} = \frac{1}{4}$ for every question. In this case, we wish to incentivize the worker to change her answer for every question. However, strong-no-free-lunch mandates that $f(\mathbf{x}) = 0$ for every $\mathbf{x} \in \{+\mathfrak{C}, -\mathfrak{C}\}^G$, and consequently, the worker will necessarily be paid a zero amount under such an action. Since any other action will also fetch an amount no less than zero, the worker is not incentivized to change her answers as required. Consequently, the strong-no-free lunch is too strong for the existence of any incentive-compatible mechanism.