

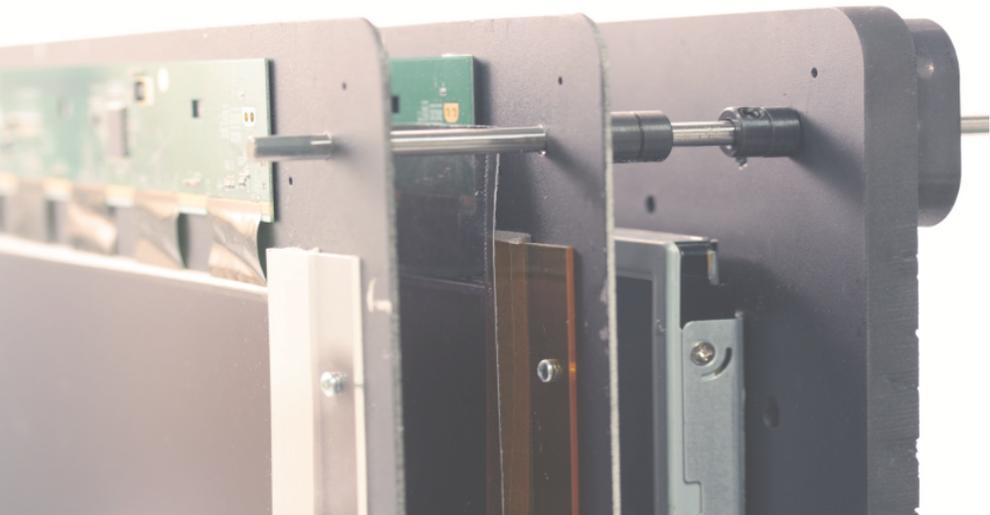
Single-pixel Imaging and ADMM

EE367/CS448I: Computational Imaging

stanford.edu/class/ee367

Lecture 11

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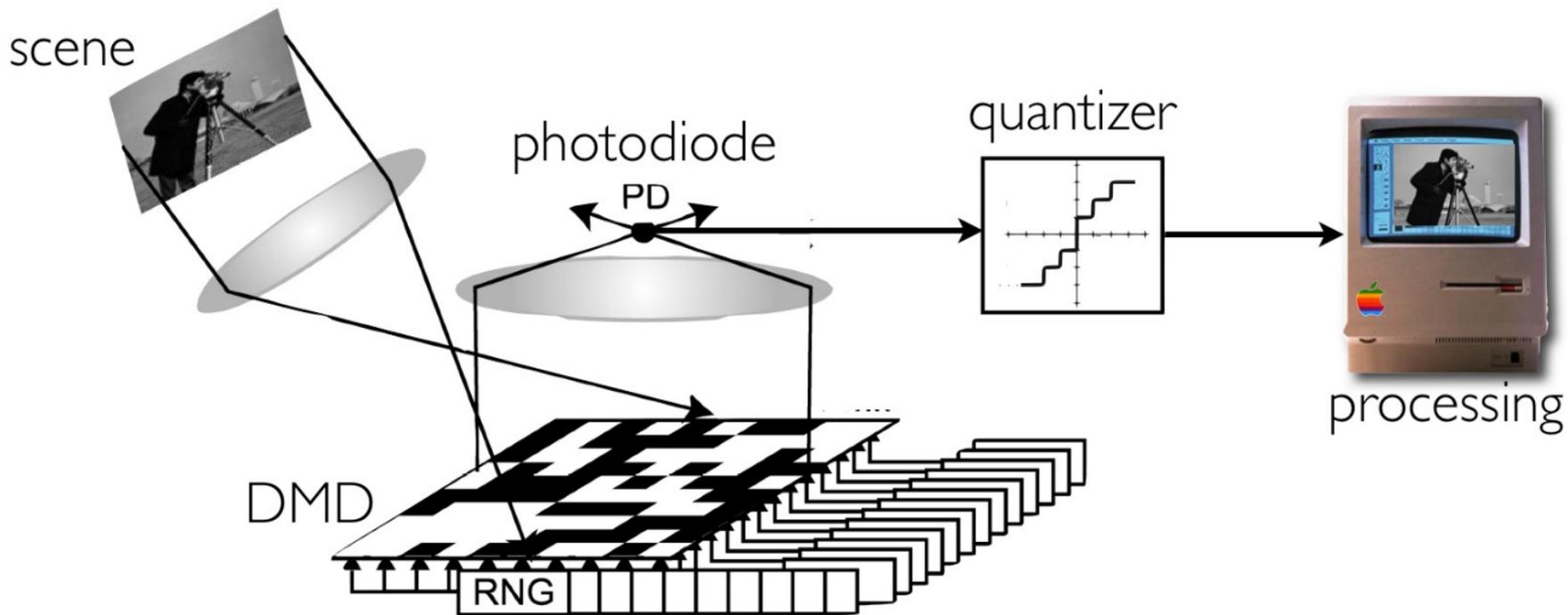


Overview

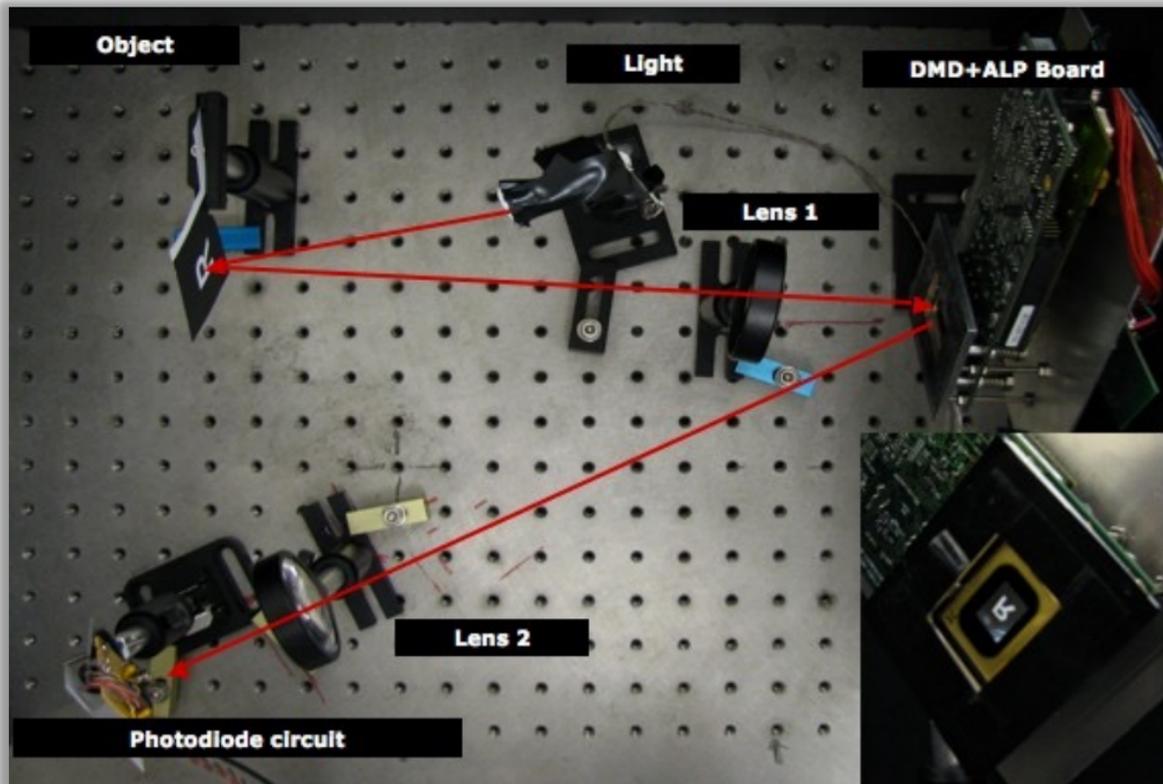
- Single-pixel imaging and other compressive imaging problems
- HQS for general inverse problems & compressive imaging
- The Alternating Direction Methods of Multipliers (ADMM)
- ADMM for general inverse problems & compressive imaging
- Outlook on using ADMM with Poisson noise and multiple regularizers

Must read: course notes on Solving Regularized Inverse Problems with ADMM!

Single-pixel Imaging



Single-pixel Imaging



original



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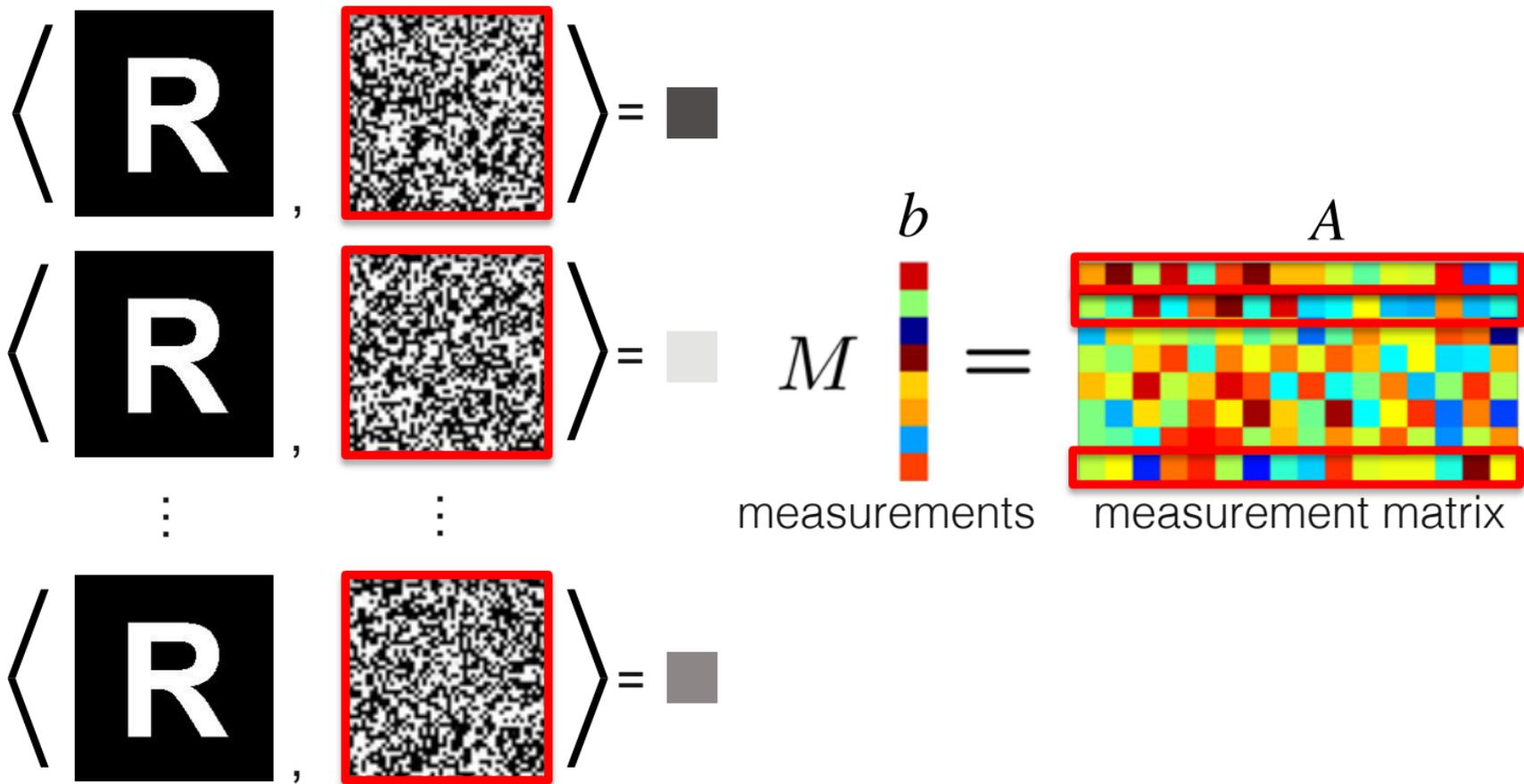


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Single-pixel Imaging



Under-determined Inverse Problems

- Image formation model: $\mathbf{b} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$
- What makes it under-determined (or a compressive imaging problem):
 $M < N$
- Problem: infinitely many solutions satisfy the observations!
Same problem as ill-posed problems! \rightarrow need **image priors**

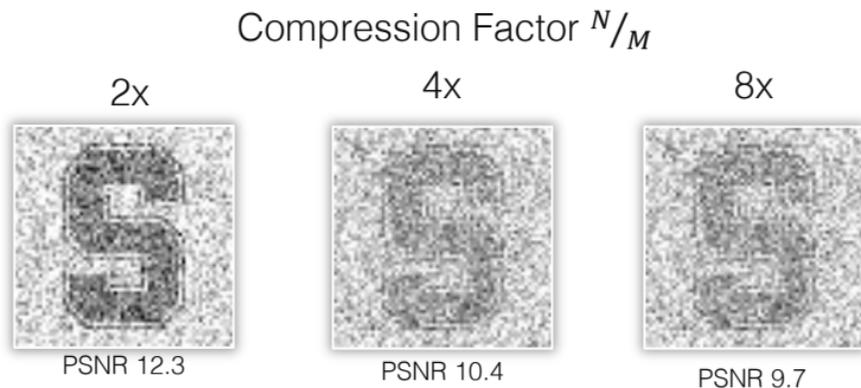
Under-determined Inverse Problems

- Image formation model: $\mathbf{b} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$
- Standard approach – the least-norm solution: $\tilde{\mathbf{x}}_{\text{ln}} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{b}$
- This is the solution of optimization problem
minimize _{\mathbf{x}} $\|\mathbf{x}\|_2$
subject to $\mathbf{A}\mathbf{x} = \mathbf{b}$

Note: among the infinitely many solutions satisfying the observations, the least-norm solution is the one with the smallest L2 norm, thus equivalent to $\|\cdot\|_2$ regularizer

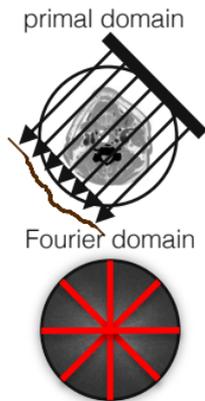
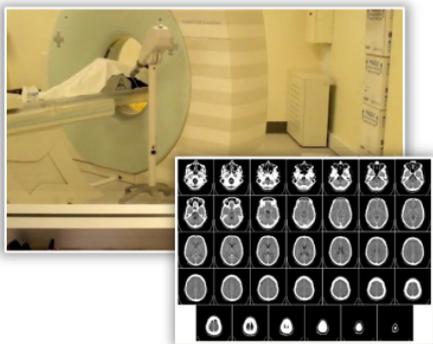
Under-determined Inverse Problems

- Image formation model: $\mathbf{b} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$
- Standard approach – the least-norm solution: $\tilde{\mathbf{x}}_{\text{ln}} = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{b}$
- Results (not great):

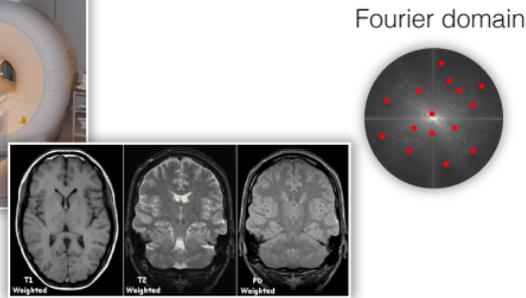


Other Inverse Problems in Imaging

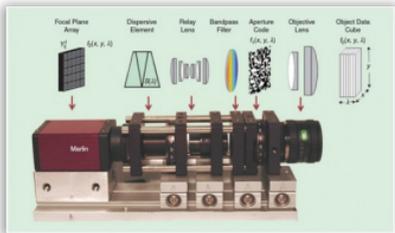
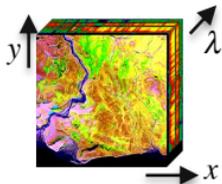
Images: Wikipedia



Computed Tomography (CT)



Magnetic Resonance Imaging (MRI)



Hyperspectral Imaging

- Computational photography
- Light field imaging
- Thermal imaging
- ...

Other Inverse Problems in Imaging

- All these inverse problems have important applications and are very different
- Yet, they all boil down to the same inverse problem, each with a different matrix \mathbf{A} :
$$\text{minimize}_{\mathbf{x}} \frac{1}{2} \|\mathbf{b} - \mathbf{Ax}\|_2^2 + \lambda\Psi(\mathbf{x})$$
- The methods derived here also apply to all those problems and applications; single-pixel imaging is a great example problem \rightarrow “if you can solve this, you can solve anything”

Review of HQS for General Inverse Problems

- Objective or “loss” function of general inverse problem:
$$\text{minimize}_x \frac{1}{2} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda \Psi(\mathbf{D}\mathbf{x})$$

↑
weight of regularizer

- Reformulate as:
$$\text{minimize}_{\{x,z\}} \underbrace{\frac{1}{2} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2}_{f(x)} + \underbrace{\lambda \Psi(\mathbf{z})}_{g(z)}$$

subject to $\mathbf{D}\mathbf{x} - \mathbf{z} = \mathbf{0}$

- Remove constraints using penalty term (equivalent for large ρ):
$$L_\rho(\mathbf{x}, \mathbf{z}) = f(\mathbf{x}) + g(\mathbf{z}) + \underbrace{\frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2}_{\text{penalty term}}$$

Review of HQS for General Inverse Problems

$$L_\rho(\mathbf{x}, \mathbf{z}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

- Alternating gradient descent approach to solving penalty formulation leads to following iterative algorithm:

while not converged:

$$\mathbf{x} \leftarrow \text{prox}_{f,\rho}(\mathbf{z}) = \arg \min_{\mathbf{x}} L_\rho(\mathbf{x}, \mathbf{z}) = \arg \min_{\mathbf{x}} f(\mathbf{x}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

$$\mathbf{z} \leftarrow \text{prox}_{g,\rho}(\mathbf{D}\mathbf{x}) = \arg \min_{\mathbf{z}} L_\rho(\mathbf{x}, \mathbf{z}) = \arg \min_{\mathbf{z}} g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

Review of HQS for General Inverse Problems

$$L_\rho(\mathbf{x}, \mathbf{z}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

\downarrow \downarrow

$$L_\rho(\mathbf{x}, \mathbf{z}) = \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \lambda\Psi(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

$\mathbf{x} \in \mathbb{R}^N$ unknown image

$\mathbf{A} \in \mathbb{R}^{M \times N}$ matrix describing image formation model

$\mathbf{z} \in \mathbb{R}^{2N}, \mathbf{D} = \begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \end{bmatrix} \in \mathbb{R}^{2N \times N}$ for TV regularizer

$\mathbf{z} \in \mathbb{R}^N, \mathbf{D} = \mathbf{I} \in \mathbb{R}^{N \times N}$ for denoising or other regularizers

Review of HQS for General Inverse Problems

\mathbf{x} – update:

$$\mathbf{x} \leftarrow \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{z}) = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

$$\mathbf{x} \leftarrow \underbrace{(\mathbf{A}^T \mathbf{A} + \rho \mathbf{D}^T \mathbf{D})}_{\tilde{\mathbf{A}}}^{-1} \underbrace{(\mathbf{A}^T \mathbf{b} + \rho \mathbf{D}^T \mathbf{z})}_{\tilde{\mathbf{b}}}$$

- For general inverse problems, we don't necessarily have an efficient closed-form solution for this problem, like we did for the deconvolution problem in lecture 10
- Use matrix-free iterative solver, such as the conjugate gradient method, to solve $\tilde{\mathbf{A}}\mathbf{x} = \tilde{\mathbf{b}}$ (e.g., `scipy.sparse.linalg.cg`)

Review of HQS for General Inverse Problems

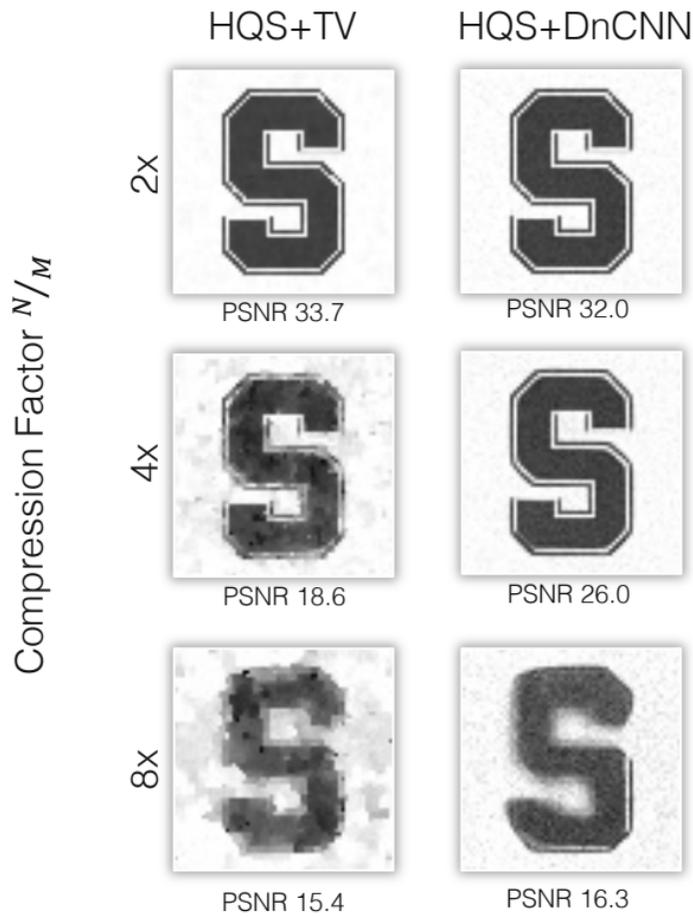
\mathbf{z} – update for TV regularizer in closed form:

$$\mathbf{z} \leftarrow \text{prox}_{\|\cdot\|_1, \rho}(\mathbf{D}\mathbf{x}) = \arg \min_{\mathbf{z}} \lambda \|\mathbf{z}\|_1 + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2 = \mathcal{S}_\kappa(\mathbf{v})$$

\mathbf{z} – update for denoising-based regularizer in closed form:

$$\mathbf{z} \leftarrow \text{prox}_{\mathcal{D}, \rho}(\mathbf{x}) = \arg \min_{\mathbf{z}} \lambda \Psi(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{x} - \mathbf{z}\|_2^2 = \mathcal{D}\left(\mathbf{x}, \sigma^2 = \frac{\lambda}{\rho}\right)$$

HQS for Single-pixel Imaging



- Works okay for low compression factor, i.e., when M is close to N
- Not very robust for larger compression factors
- Formulation using penalty term is not adequate \rightarrow need something more robust

HQS vs. ADMM

- Objective function:
$$\text{minimize}_{\mathbf{x}} \frac{1}{2} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\Psi(\mathbf{D}\mathbf{x})$$

- Reformulate as:
$$\text{minimize}_{\{\mathbf{x}, \mathbf{z}\}} \underbrace{\frac{1}{2} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2}_{f(\mathbf{x})} + \underbrace{\lambda\Psi(\mathbf{z})}_{g(\mathbf{z})}$$

subject to $\mathbf{D}\mathbf{x} - \mathbf{z} = \mathbf{0}$

- Penalty Method of HQS:
$$L_{\rho}^{(\text{HQS})}(\mathbf{x}, \mathbf{z}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2$$

- Augmented Lagrangian:
$$\begin{aligned} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{y}) &= f(\mathbf{x}) + g(\mathbf{z}) + \mathbf{y}^T(\mathbf{D}\mathbf{x} - \mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z}\|_2^2 \\ &\stackrel{u = (1/\rho)\mathbf{y}}{=} f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2 \end{aligned}$$

ADMM

$$L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2$$

$\mathbf{x} \in \mathbb{R}^N$ unknown image

$\mathbf{A} \in \mathbb{R}^{M \times N}$ matrix describing image formation model

$\mathbf{z}, \mathbf{u} \in \mathbb{R}^{2N}, \mathbf{D} = \begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \end{bmatrix} \in \mathbb{R}^{2N \times N}$ for TV regularizer

$\mathbf{z}, \mathbf{u} \in \mathbb{R}^N, \mathbf{D} = \mathbf{I} \in \mathbb{R}^{N \times N}$ for denoising or other regularizers

ADMM

$$L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2$$

- Alternating gradient descent approach to solving Augmented Lagrangian:

while not converged:

$$\mathbf{x} \leftarrow \text{prox}_{f, \rho}(\mathbf{z}) = \arg \min_{\mathbf{x}} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{x}} f(\mathbf{x}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

$$\mathbf{z} \leftarrow \text{prox}_{g, \rho}(\mathbf{D}\mathbf{x}) = \arg \min_{\mathbf{z}} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{z}} g(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

$$\mathbf{u} \leftarrow \mathbf{u} + \mathbf{D}\mathbf{x} - \mathbf{z}$$

ADMM

\mathbf{x} - update:

$$\mathbf{x} \leftarrow \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{z}) = \arg \min_x \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + \frac{\rho}{2} \|\mathbf{D}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2,$$

$$\mathbf{x} \leftarrow \underbrace{(\mathbf{A}^T \mathbf{A} + \rho \mathbf{D}^T \mathbf{D})}_{\tilde{\mathbf{A}}}^{-1} \underbrace{(\mathbf{A}^T \mathbf{b} + \rho \mathbf{D}^T (\mathbf{z} - \mathbf{u}))}_{\tilde{\mathbf{b}}}$$

- Same general \mathbf{x} -update as HQS, use matrix-free iterative solver, such as the conjugate gradient method, to solve $\tilde{\mathbf{A}}\mathbf{x} = \tilde{\mathbf{b}}$ (e.g., `scipy.sparse.linalg.cg`)

ADMM

\mathbf{z} – update for TV regularizer in closed form:

$$\mathbf{z} \leftarrow \text{prox}_{\|\cdot\|_1, \rho}(\mathbf{v}) = \arg \min_{\mathbf{z}} \lambda \|\mathbf{z}\|_1 + \frac{\rho}{2} \|\mathbf{v} - \mathbf{z}\|_2^2 = \mathcal{S}_{\kappa}(\mathbf{v}), \mathbf{v} = \mathbf{D}\mathbf{x} + \mathbf{u}$$

\mathbf{z} – update for denoising-based regularizer in closed form:

$$\mathbf{z} \leftarrow \text{prox}_{\mathcal{D}, \rho}(\mathbf{x} + \mathbf{u}) = \arg \min_{\mathbf{z}} \lambda \Psi(\mathbf{z}) + \frac{\rho}{2} \|\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 = \mathcal{D}\left(\mathbf{x} + \mathbf{u}, \sigma^2 = \frac{\lambda}{\rho}\right)$$

→ Same \mathbf{z} -update rules as HQS!

ADMM

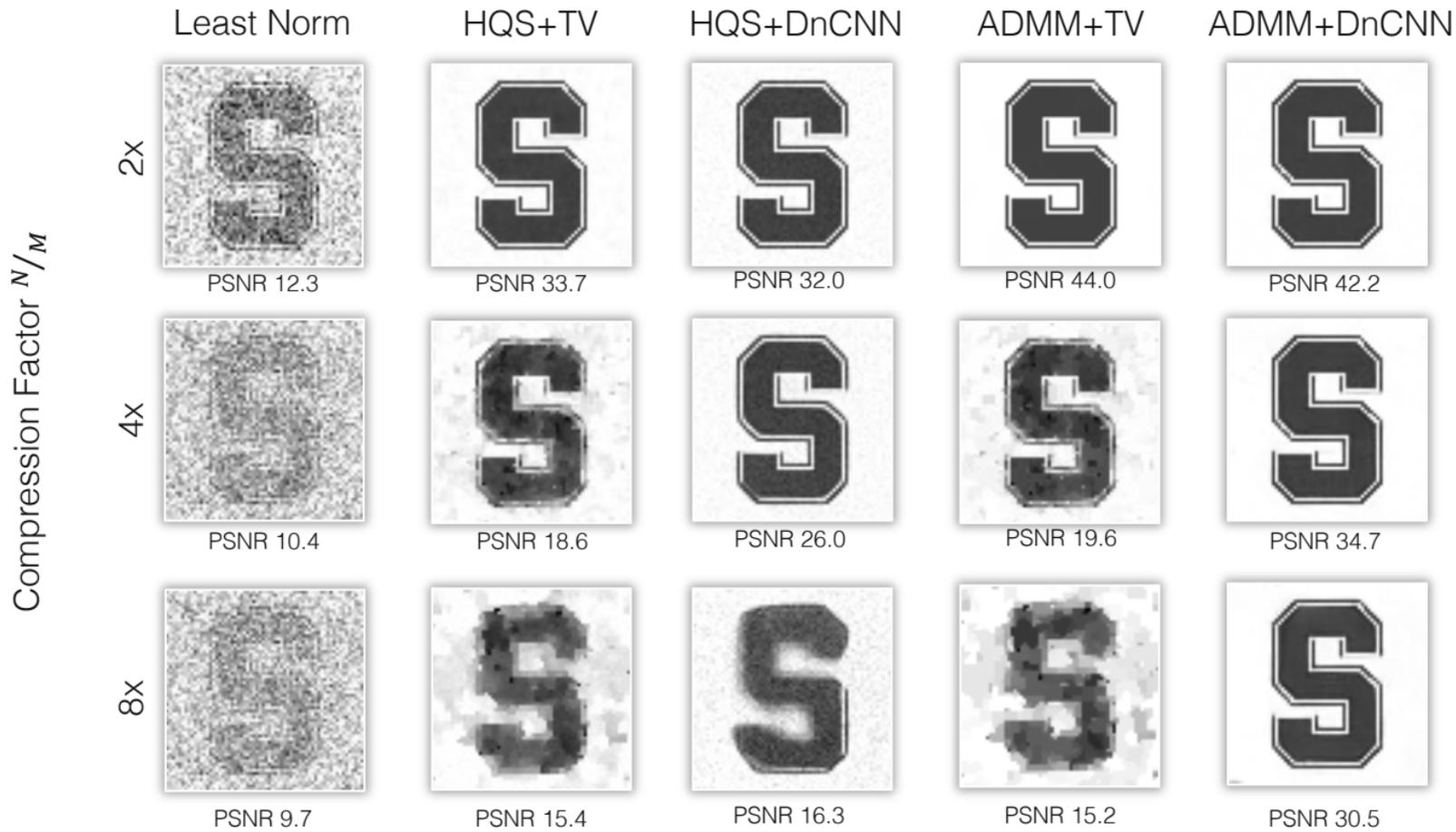
ADMM for inverse problem with denoiser

```
1: initialize  $\rho$  and  $\lambda$ 
2:  $\mathbf{x} = \text{zeros}(W, H)$ ;
3:  $\mathbf{z} = \text{zeros}(W, H)$ ;
4:  $\mathbf{u} = \text{zeros}(W, H)$ ;
5: for  $k = 1$  to  $\text{max\_iters}$  do
6:    $\mathbf{x} = \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{v}) = \text{cg\_solve}(\mathbf{A}^T \mathbf{A} + \rho \mathbf{I}, \mathbf{A}^T \mathbf{b} + \rho(\mathbf{z} - \mathbf{u}))$ 
7:    $\text{prox}_{\mathcal{D}, \rho}(\mathbf{x} + \mathbf{u}) = \mathcal{D}\left(\mathbf{x} + \mathbf{u}, \sigma^2 = \frac{\lambda}{\rho}\right)$ 
8:    $\mathbf{u} = \mathbf{u} + \mathbf{x} - \mathbf{z}$ 
9: end for
```

ADMM for inverse problem with TV

```
1: initialize  $\rho$  and  $\lambda$ 
2:  $\mathbf{x} = \text{zeros}(W, H)$ ;
3:  $\mathbf{z} = \text{zeros}(W, H, 2)$ ;
4:  $\mathbf{u} = \text{zeros}(W, H, 2)$ ;
5: for  $k = 1$  to  $\text{max\_iters}$  do
6:    $\mathbf{x} = \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{z} - \mathbf{u}) = \text{cg\_solve}(\mathbf{A}^T \mathbf{A} + \rho \mathbf{D}^T \mathbf{D}, \mathbf{A}^T \mathbf{b} + \rho \mathbf{D}^T(\mathbf{z} - \mathbf{u}))$ 
7:    $\mathbf{z} = \text{prox}_{\|\cdot\|_1, \rho}(\mathbf{D}\mathbf{x} + \mathbf{u}) = \mathcal{S}_{\lambda/\rho}(\mathbf{D}\mathbf{x} + \mathbf{u})$ 
8:    $\mathbf{u} = \mathbf{u} + \mathbf{D}\mathbf{x} - \mathbf{z}$ 
9: end for
```

ADMM – Results



Back to the Bayesian Perspective of Inverse Problems

Note: the following material is optional and
not part of any homework or the midterm!

Bayesian Perspective of Gaussian Noise

- Image formation model: $\mathbf{b} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$

- Joint probability of all observations:
$$p(\mathbf{b}|\mathbf{x}, \sigma) = \prod_{i=1}^M p(\mathbf{b}_i|\mathbf{x}_i, \sigma) \propto e^{-\frac{\|\mathbf{b}-\mathbf{A}\mathbf{x}\|_2^2}{2\sigma^2}}$$

- Bayes' rule:
$$p(\mathbf{x}|\mathbf{b}, \sigma) = \frac{p(\mathbf{b}|\mathbf{x}, \sigma)p(\mathbf{x})}{p(\mathbf{b})} \propto p(\mathbf{b}|\mathbf{x}, \sigma)p(\mathbf{x})$$

- Maximum-a-posterior (MAP) solution:

$$\begin{aligned}\mathbf{x}_{MAP} &= \arg \min_{\mathbf{x}} -\log(p(\mathbf{x}|\mathbf{b}, \sigma)) \\ &= \arg \min_{\mathbf{x}} \frac{1}{2\sigma^2} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 + \Psi(\mathbf{x})\end{aligned}$$

Bayesian Perspective of Poisson Noise

- Image formation model: $\mathbf{b} = \mathcal{P}(\mathbf{Ax})$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$

- Probability of observation i :
$$p(\mathbf{b}_i | \mathbf{x}) = \frac{(\mathbf{Ax})_i^{b_i} e^{-(\mathbf{Ax})_i}}{b_i!}$$

- Joint probability of all observations:
$$p(\mathbf{b} | \mathbf{x}) = \prod_{i=1}^M p(\mathbf{b}_i | \mathbf{x})$$
$$= \prod_{i=1}^M e^{\log((\mathbf{Ax})_i) b_i} \cdot e^{-(\mathbf{Ax})_i} \cdot \frac{1}{b_i!}$$

Bayesian Perspective of Poisson Noise

- Image formation model: $\mathbf{b} = \mathcal{P}(\mathbf{A}\mathbf{x})$, $\mathbf{b} \in \mathbb{R}^M$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{A} \in \mathbb{R}^{M \times N}$

- Bayes' rule:
$$p(\mathbf{x}|\mathbf{b}, \sigma) = \frac{p(\mathbf{b}|\mathbf{x}, \sigma)p(\mathbf{x})}{p(\mathbf{b})} \propto p(\mathbf{b}|\mathbf{x}, \sigma)p(\mathbf{x})$$

- Maximum-a-posterior (MAP) solution:

$$\begin{aligned}\mathbf{x}_{MAP} &= \arg \min_{\mathbf{x}} -\log(p(\mathbf{x}|\mathbf{b}, \sigma)) = -\log(p(\mathbf{b}|\mathbf{x})) - \log(p(\mathbf{x})) \\ &= \arg \min_{\mathbf{x}} -\log(p(\mathbf{b}|\mathbf{x})) + \lambda\Psi(\mathbf{x})\end{aligned}$$

ADMM+TV for Poisson Noise & Nonnegativity

- Objective function:
$$\underset{\mathbf{x}}{\text{minimize}} \quad -\log(p(\mathbf{b}|\mathbf{x})) + \lambda\Psi(\mathbf{D}\mathbf{x})$$

does not include A
includes A
- Reformulate as:
$$\underset{\{x,z\}}{\text{minimize}} \quad \underbrace{-\log(p(\mathbf{b}|\mathbf{z}_1))}_{g_1(\mathbf{z}_1)} + \underbrace{\lambda_1\|\mathbf{z}_2\|_1}_{g_2(\mathbf{z}_2)} + \underbrace{\mathcal{J}_{\mathbb{R}_+}(\mathbf{z}_3)}_{g_3(\mathbf{z}_3)}$$
- Indicator function:
$$\mathcal{J}_{\mathbb{R}_+}(v) = \begin{cases} 0 & v > 0 \\ \infty & \text{otherwise} \end{cases}$$

subject to
$$\underbrace{\begin{bmatrix} \mathbf{A} \\ \mathbf{D} \\ \mathbf{I} \end{bmatrix}}_{\mathbf{K}} \mathbf{x} - \underbrace{\begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \end{bmatrix}}_{\mathbf{z}} = \mathbf{0}$$
- Scaled Augmented Lagrangian:
$$L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \sum_i g_i(\mathbf{z}_i) + \frac{\rho}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2$$

ADMM+TV for Poisson Noise & Nonnegativity

$$L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \sum_i g_i(\mathbf{z}_i) + \frac{\rho}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2} \|\mathbf{u}\|_2^2$$

- Alternating gradient descent approach to solving
Augmented Lagrangian:

while not converged:

$$\mathbf{x} \leftarrow \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{z}) = \arg \min_{\mathbf{x}} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

for all i :

$$\mathbf{z}_i \leftarrow \text{prox}_{g_i, \rho}(\mathbf{x}) = \arg \min_{\mathbf{z}_i} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{z}_i} g_i(\mathbf{z}_i) + \frac{\rho}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

$$\mathbf{u} \leftarrow \mathbf{u} + \mathbf{K}\mathbf{x} - \mathbf{z}$$

ADMM+TV for Poisson Noise & Nonnegativity

- Derivation of all these proximal operators in the course notes on Noise, Denoising, and Image Reconstruction with Noise!

while not converged:

$$\mathbf{x} \leftarrow \text{prox}_{\|\cdot\|_2, \rho}(\mathbf{z}) = \arg \min_{\mathbf{x}} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

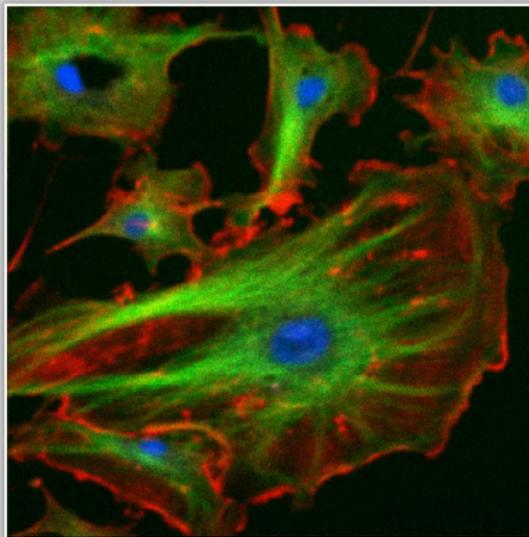
for all i :

$$\mathbf{z}_i \leftarrow \text{prox}_{g_i, \rho}(\mathbf{x}) = \arg \min_{\mathbf{z}_i} L_{\rho}^{(\text{ADMM})}(\mathbf{x}, \mathbf{z}, \mathbf{u}) = \arg \min_{\mathbf{z}_i} g_i(\mathbf{z}_i) + \frac{\rho}{2} \|\mathbf{K}\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2$$

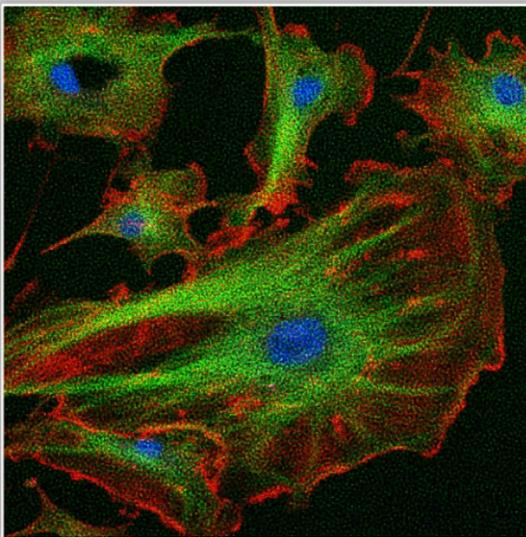
$$\mathbf{u} \leftarrow \mathbf{u} + \mathbf{K}\mathbf{x} - \mathbf{z}$$

ADMM+TV for Poisson Noise & Nonnegativity

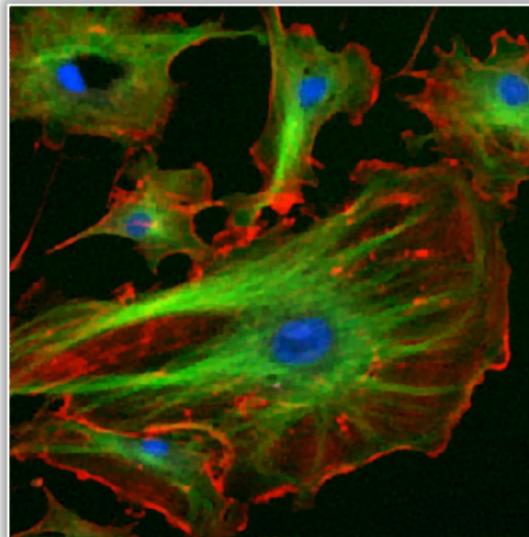
Blurry & Noisy Measurements



Richardson-Lucy Method
(maximum likelihood solution)



ADMM+TV+Nonnegativity
(maximum-a-posteriori solution)



References and Further Reading

Must read: EE367 course notes on Solving Regularized Inverse Problems with ADMM!

Optional read: EE367 course notes on Noise, Denoising, and Image Reconstruction with Noise

ADMM

- S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein “Distributed optimization and statistical learning via the alternating direction method of multipliers”, Foundation and Trends in Machine Learning, 2001

Single-pixel Imaging

- M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, R. Baraniuk “Single-pixel imaging via compressive sampling”, IEEE Signal Processing Magazine 2008