Introduction to Wave Optics and Deep Optics

EE367/CS448I: Computational Imaging

stanford.edu/class/ee367

Lecture 14

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Overview & Motivation

Wave optics in a nutshell: coherence, Hyugens-Fresnel principle, wave propagation, wave field vs observable intensity, diffraction-limited resolution, …

Goals:

- Intuitive introduction of fundamentals of wave optics without all the math (not enough time, take an optics class for that)
- Overview of modern approaches combining wave optics with artificial intelligence techniques for various applications

Coherence in a Nutshell

- Incoherent light: emission of light over "large" (i.e., compared to wavelength) area & broad range of wavelengths

- Partially coherent light (one of the following):
  - point or plane wave = spatially coherent
  - monochromatic = temporally coherent

- Coherent light: spatially & temporally coherent

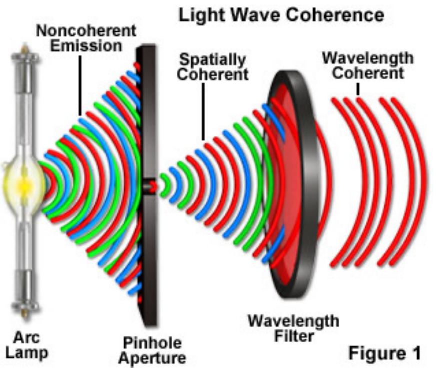
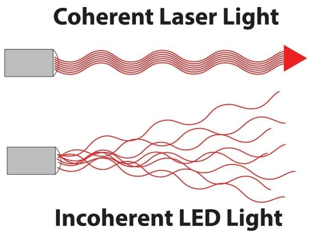


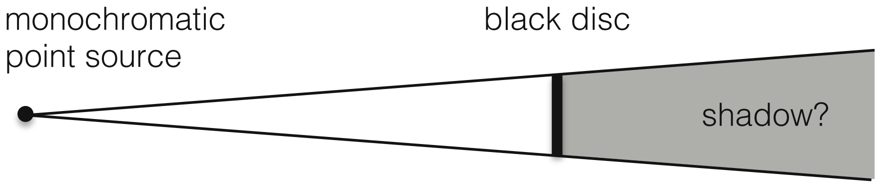
Image courtesy: Zeiss



https://www.informationpalace.com/light-amplification-by-stimulated-emission-of-radiation/

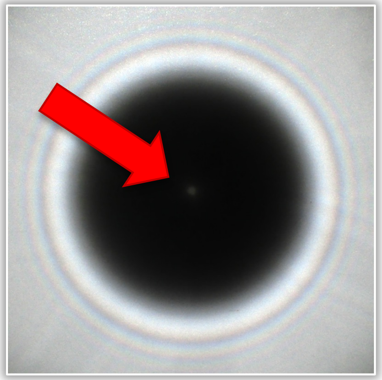
Poisson's Spot

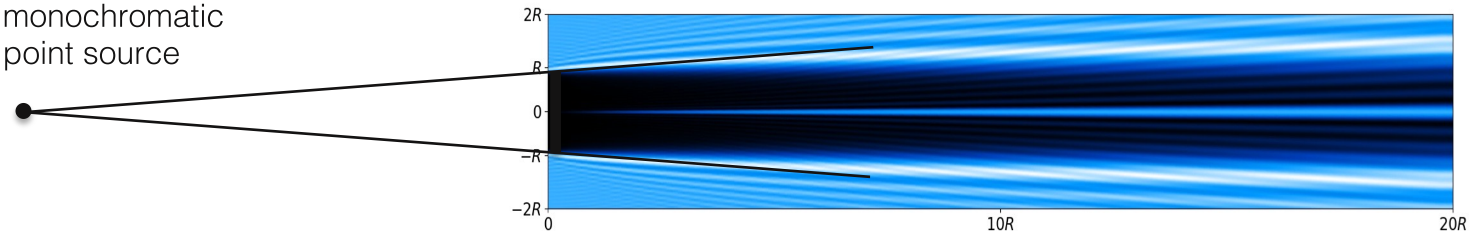
- Common sense says there's a shadow behind an occluder, like a disc
- Wave theory predicts bright spot



Poisson’s Spot

- Common sense says there’s a shadow behind an occluder, like a disc
- Wave theory predicts bright spot
- Fresnel predicted, Poisson doubted, and Arago demonstrated it in 1818

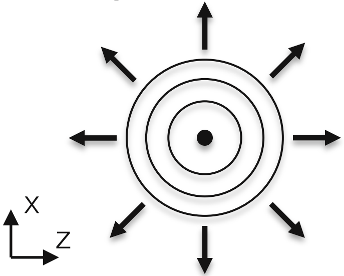




Point Sources and Plane Waves

- Point source at $\boldsymbol{x}_0$ and amplitude u_0
- $r = \|\boldsymbol{x} - \boldsymbol{x}_0\|_2, k = \frac{2\pi}{\lambda}$

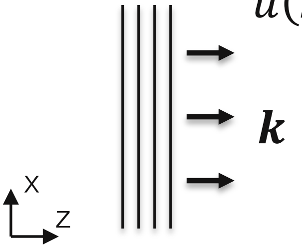
$$u(\boldsymbol{x}) = u_0 \frac{e^{ikr}}{r}$$



point source

- Propagates with velocity c into direction $\boldsymbol{k} = (k_x, k_y, k_z)$
- "what comes out of a laser" or collimated point source

$$u(\boldsymbol{x}) = u_0 e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$



plane wave

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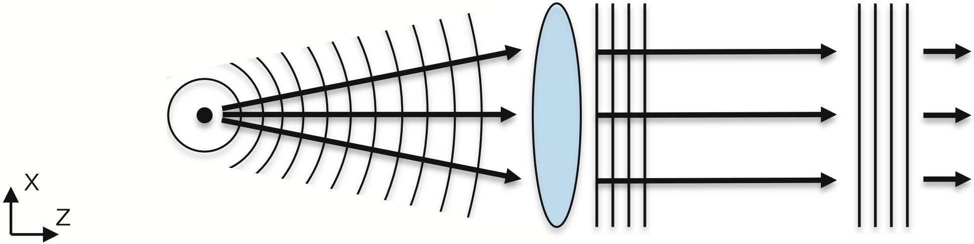


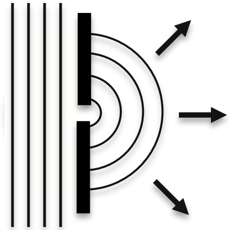
point source

plane wave

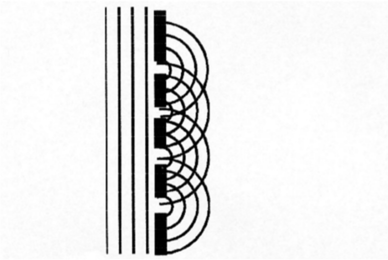
Hyugens-Fresnel Principle

Every point on a wavefront is itself the source of spherical wavelets, and the secondary wavelets emanating from different points mutually interfere. The sum of these spherical wavelets forms the wavefront.

Examples:

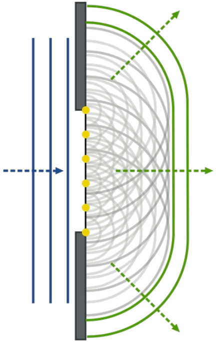


plane wave through slit → spherical wave

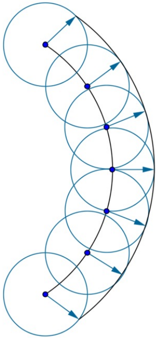


plane wave through multiple slits → interfering sph. waves

sph. waves make plane wave



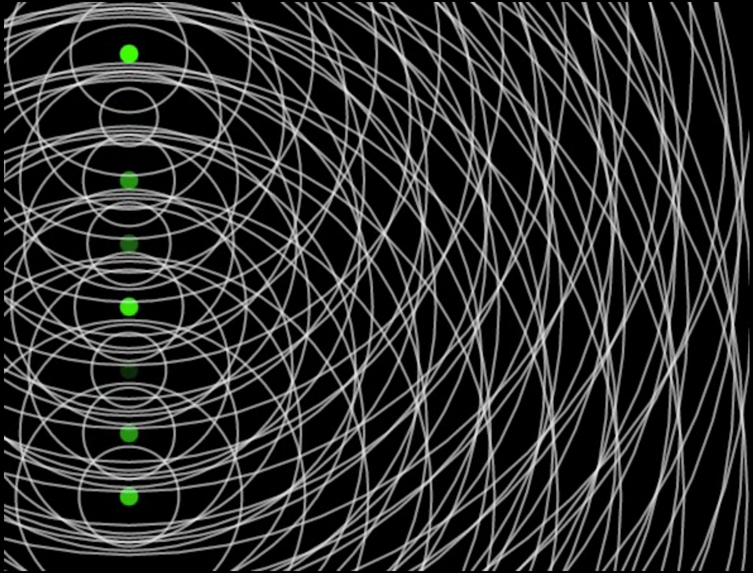
plane wave through big slit



curved wave

Hyugens-Fresnel Principle

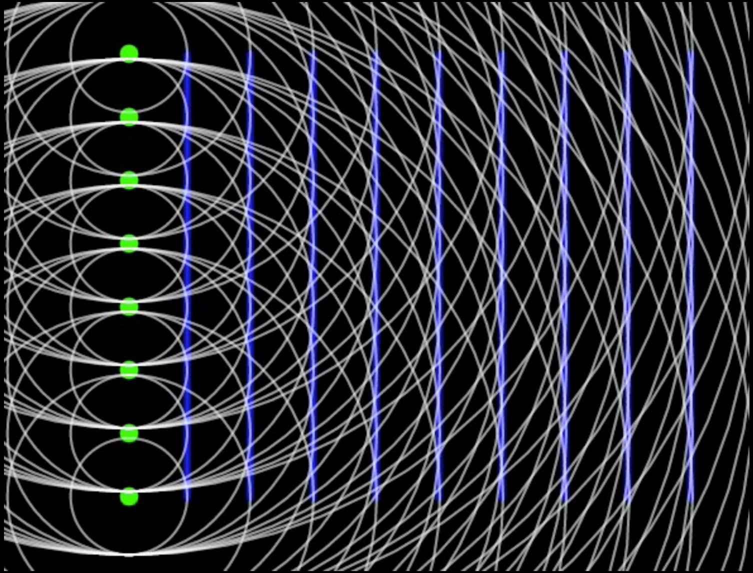
Direction: random



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

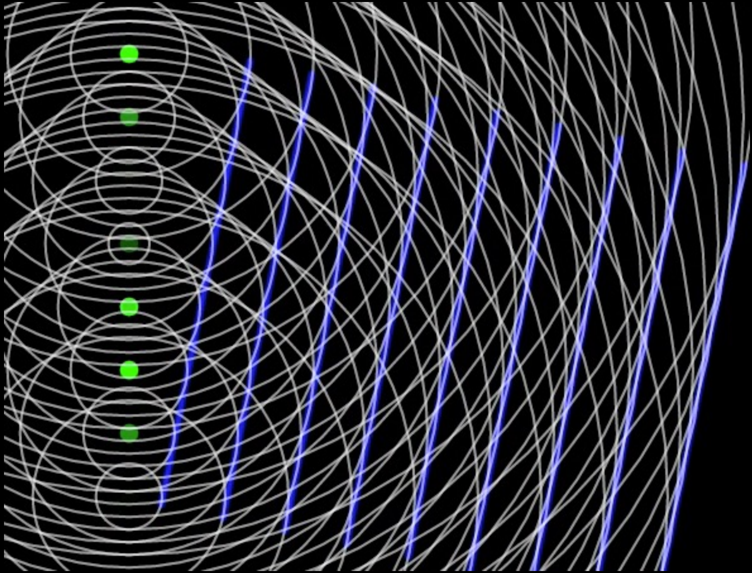
Direction: axial



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

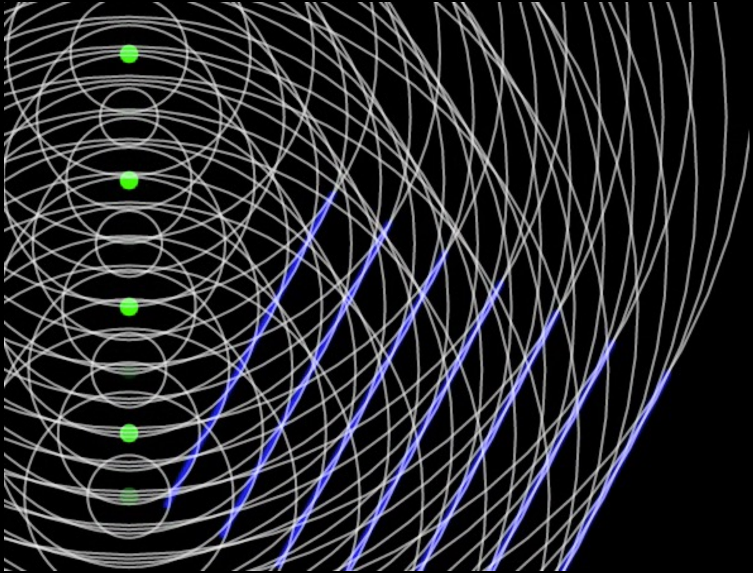
Direction: oblique



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

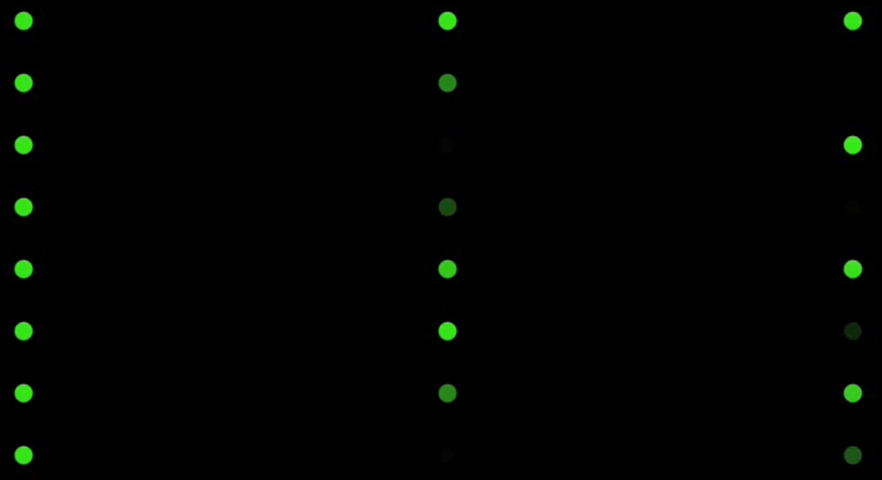
Direction: more oblique



images/videos from [Zhang & Levoy, 2009]

Spatial Frequency

axial | oblique | more oblique



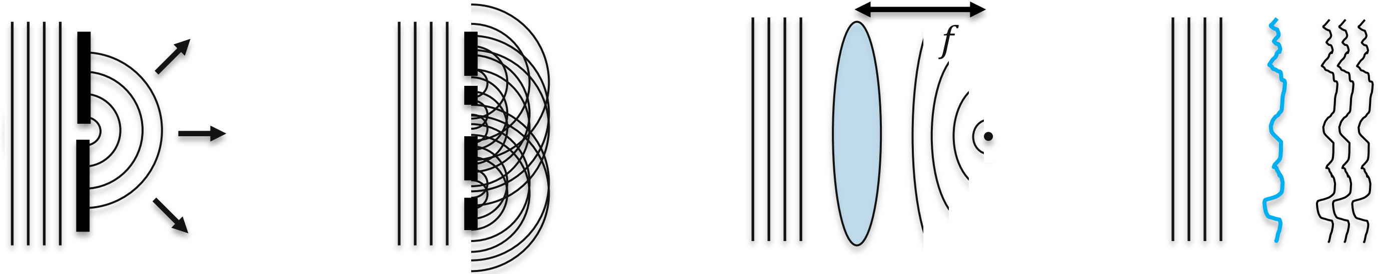
zero spatial frequency | low spatial frequency | higher spatial frequency

images/videos from [Zhang & Levoy, 2009]

Interaction of Wave Field and Thin Object

- Field before/after object $u_{in/out}(x)$ are defined by amplitude $a(x)$ and phase $\phi(x)$ of mask (or object) as

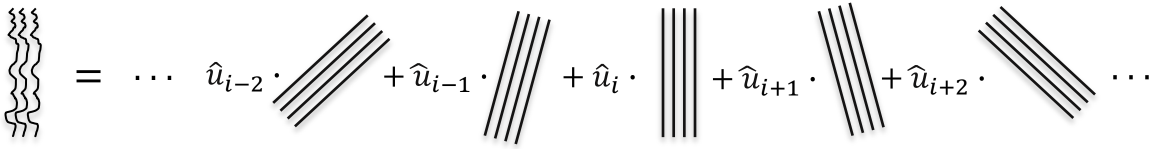
$$u_{out}(x) = u_{mask}(x)\, u_{in}(x) = a(x)e^{i\phi(x)}\, u_{in}(x)$$



slit	amplitude mask	lens	phase mask
$a(x) = \boldsymbol{\delta}(x - x_0)$ $\phi(x) = 1$	$a(x) = rand$ $\phi(x) = 1$	$a(x) = 1$ $\phi(x) = -{}^{k}/_{2f}\, x^2$	$a(x) = 1,$ $\phi(x) = rand$

Plane Wave Decomposition = Fourier Transform

- Every wavefield can be represented as a weighted sum of plane waves



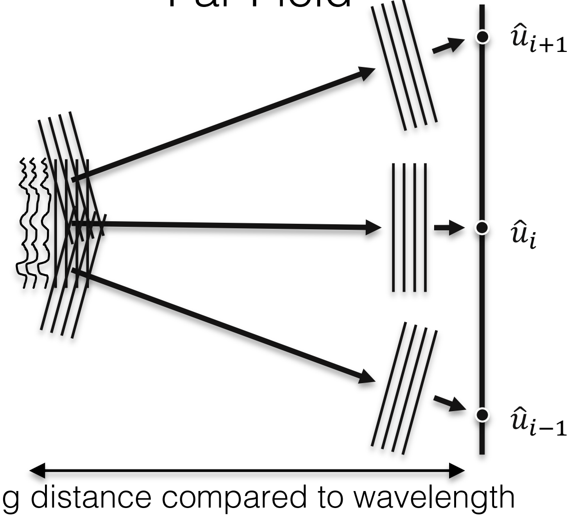
- Decomposition is the Fourier transform: $u(x) = \int \hat{u}(k_x)\, e^{2\pi i x k_x}\, dk_x$

1D plane wave with slope

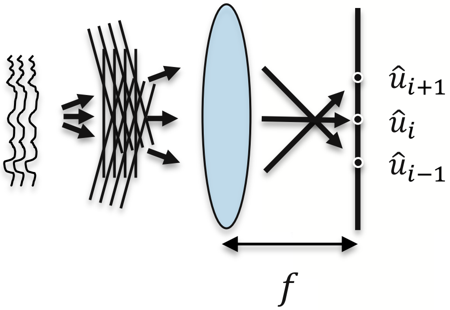
Optical Fourier Transform via Wave Propagation

Propagating plane waves in free space is intuitive → long distances in free space or lenses perform optical Fourier transform

Far Field

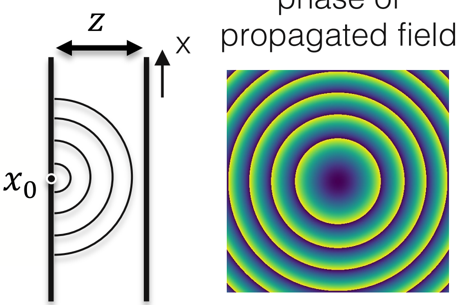


Lens



Wave Propagation in Free Space

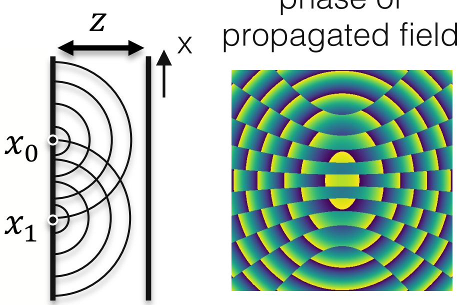
single point source



$$\mathrm{d} = \sqrt{(\boldsymbol{x} - \boldsymbol{x}_0)^2 + z^2}$$

$$u(\boldsymbol{x}) = u_0 \frac{e^{i\frac{2\pi}{\lambda}d}}{d}$$

two point sources



$$d_{0/1} = \sqrt{\left(\boldsymbol{x} - \boldsymbol{x}_{0/1}\right)^2 + z^2}$$

$$u(\boldsymbol{x}) = u_0 \frac{e^{i\frac{2\pi}{\lambda}d_0}}{d_0} + u_1 \frac{e^{i\frac{2\pi}{\lambda}d_1}}{d_1}$$

Wave Propagation in Free Space

In general, free-space propagation by distance z is modeled by convolution with a complex-valued propagation kernel or, similarly, a multiplication with a transfer function $\mathcal{H}$ in the Fourier domain.

$$u_{prop}(x,y) = \mathfrak{I}^{-1}\left\{\mathfrak{I}\{u(x',y')\} \cdot \mathcal{H}\left(k_x, k_y, z\right)\right\}$$

$$\mathcal{H}\left(k_x, k_y, z\right) = \begin{cases} e^{-i\frac{2\pi}{\lambda}\sqrt{1-(\lambda k_x)^2-\left(\lambda k_y\right)^2}\,z} & if\ \sqrt{k_x^2+k_y^2} < \frac{1}{\lambda} \\ 0 & otherwise \end{cases}$$

Different propagation operators have different transfer functions, this one is called Angular Spectrum Method (ASM).

[Goodman 2012]

Complex Field vs. Observable Intensity

We cannot directly observe the field, only its squared amplitude or intensity

observable intensity

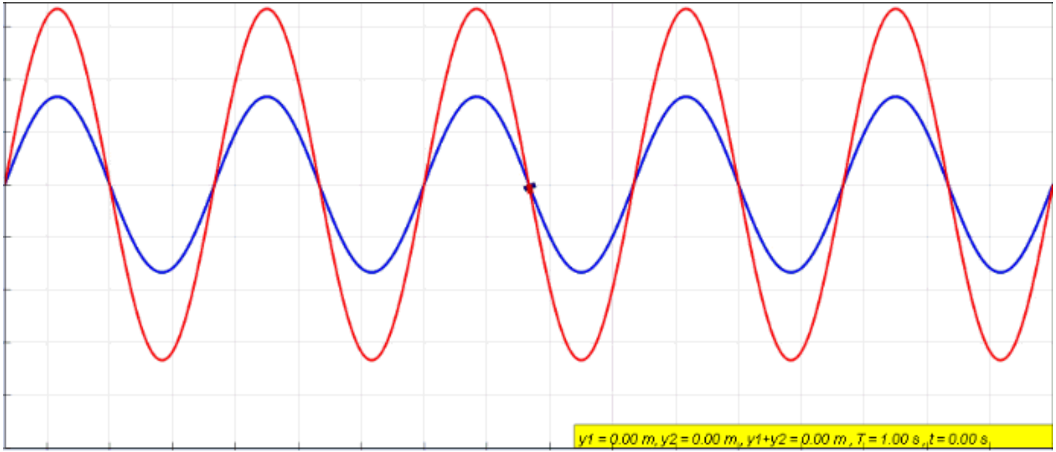
amplitude squared,
no phase information!

$$\mathrm{I}(x, y) = |u(x, y)|^2 = \left|a(x, y)e^{i\phi(x,y)}\right|^2 = a^2(x, y)$$

complex-valued field

Constructive & Destructive Interference

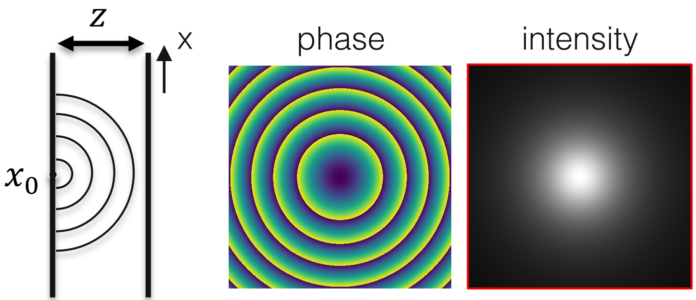
https://en.wikipedia.org/wiki/Wave_interference



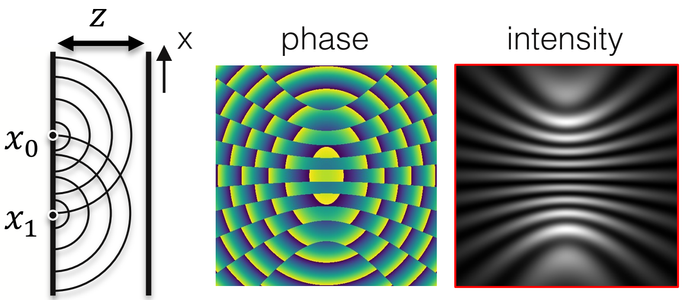
$$\left|a_1 e^{i\phi_1} + a_2 e^{i\phi_2}\right|^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\phi_1 - \phi_2)$$

Constructive & Destructive Interference

single point source



two point sources

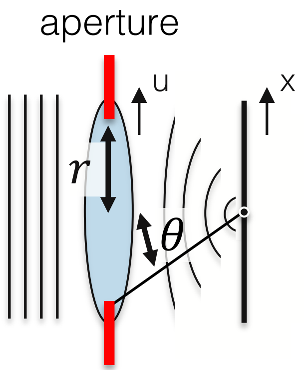


no interference

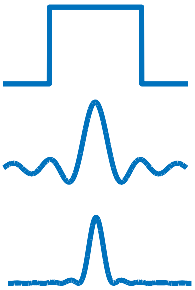
interference

$$i(\boldsymbol{x}) = |u(\boldsymbol{x})|^2$$

Diffraction-limited Resolution

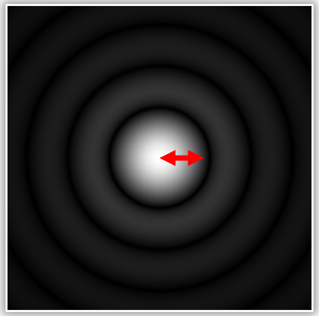


1D:
- Aperture is $rect(ru)$
- Wave at sensor is $sinc(krx)$
- Intensity is $|sinc(krx)|^2, k = \frac{2\pi}{\lambda}$



2D:
- Aperture is $circ(r)$
- Wave at sensor is
$$jinc(kr\sin\theta) = \frac{J_1(kr\sin\theta)}{kr\sin\theta}$$
- Intensity is $|jinc(kr\sin\theta)|^2$ →

Airy disk



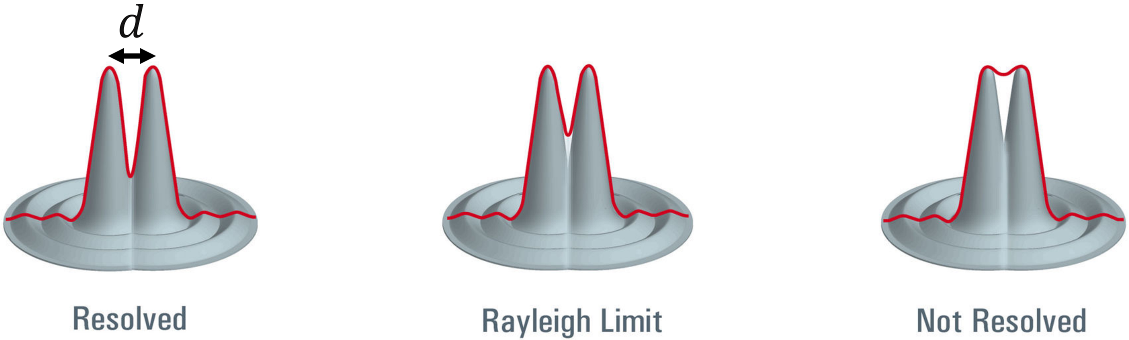
First minimum at radius

$$1.22\frac{\lambda}{2n\sin\theta}$$

$n\sin\theta$ related to f-number or NA of lens

Diffraction-limited Resolution

https://www.leica-microsystems.com/science-lab/microscope-resolution-concepts-factors-and-calculation/

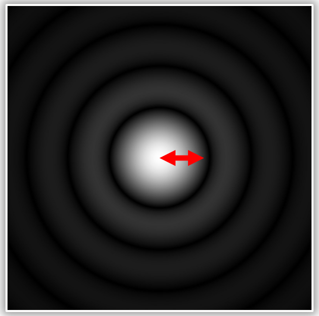


Can resolve 2 points if distance d is at least:

- Rayleigh Limit $1.22\,\dfrac{\lambda}{2n\sin\theta}$
- Abbe Limit $\dfrac{\lambda}{2n\sin\theta}$

n is refractive index of medium

Airy disk



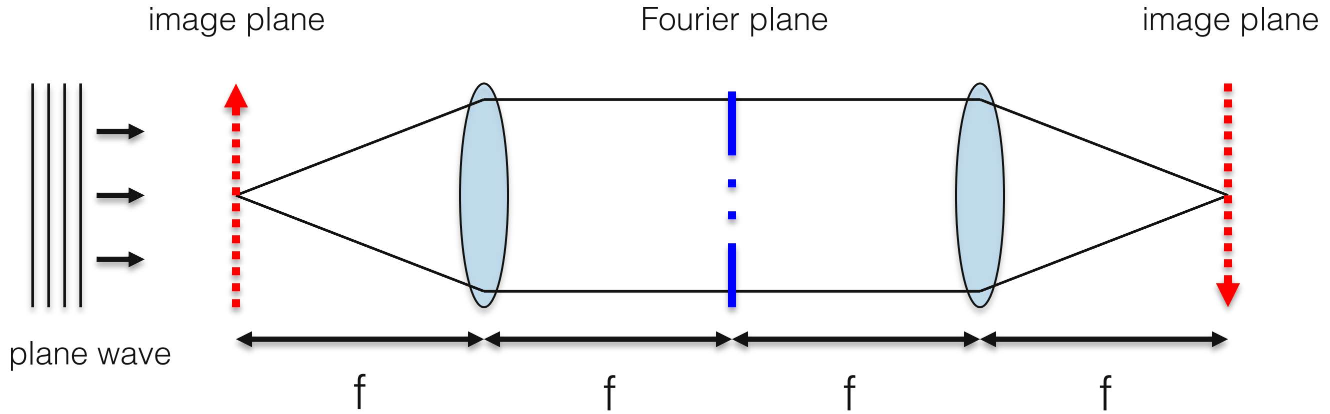
First minimum at radius

$$1.22\,\frac{\lambda}{2n\sin\theta}$$

$n\sin\theta$ related to f-number or NA of lens

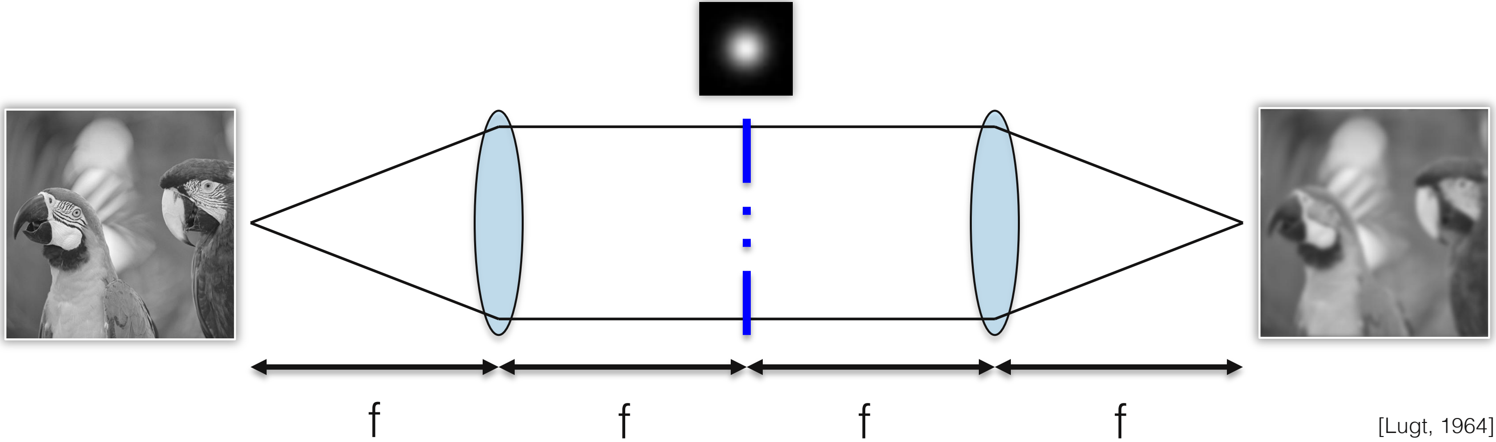
4f system

- 4f system consists of 2 lenses spaced at 2x their focal length f
- Image plane is copied at 4f distance
- Fourier plane between lenses



Optical Correlator

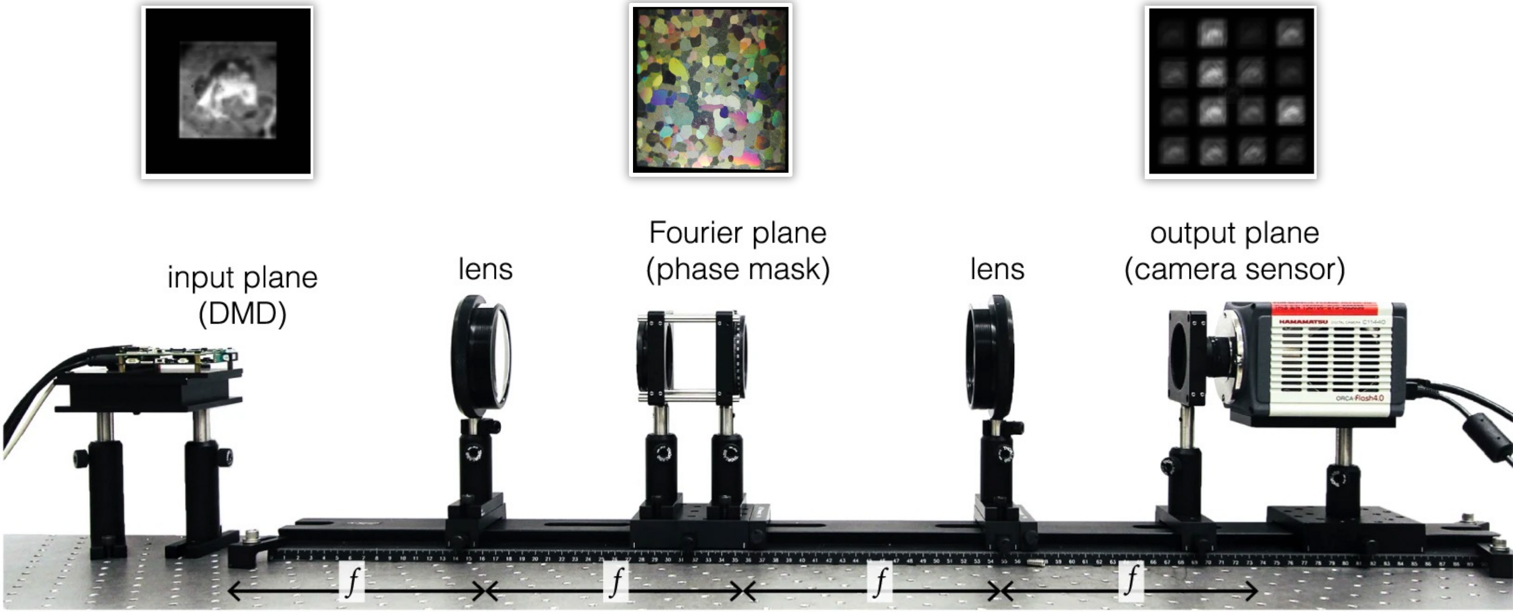
- Amplitude or phase mask placed at Fourier plane performs optical filtering / correlation
- Can implement low-, high-, or bandpass filter optically!



[Lugt, 1964]

Optical CNN

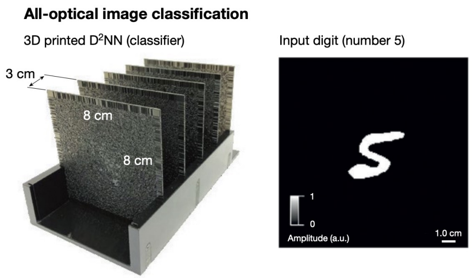
- Copy input image & convolve with different kernels = CNN



[Chang et al., 2018]

Optical Computing

Deep Diffractive Neural Networks



Confusion matrix — Experimental

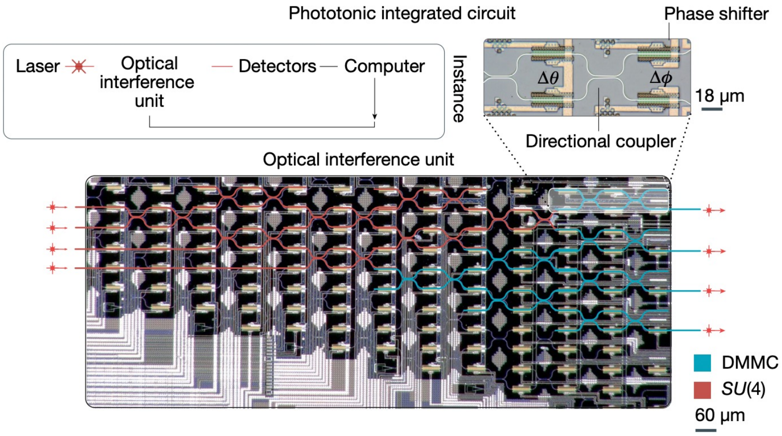
Predicted labels \ True labels	0	1	2	3	4	5	6	7	8	9
0	3	0	0	0	0	0	0	0	0	0
1	0	5	0	1	0	0	0	0	0	0
2	1	0	5	0	0	0	0	0	0	0
3	0	0	0	4	0	0	0	0	1	0
4	0	0	0	0	5	0	0	0	0	1
5	1	0	0	0	0	5	1	0	0	0
6	0	0	0	0	0	0	4	0	0	0
7	0	0	0	0	0	0	0	5	0	0
8	0	0	0	0	0	0	0	0	4	0
9	0	0	0	0	0	0	0	0	0	4

Confusion matrix — Designed

Predicted labels \ True labels	0	1	2	3	4	5	6	7	8	9
0	955	0	11	5	1	9	9	3	10	10
1	0	1121	14	1	11	5	4	26	10	11
2	1	2	889	23	2	0	2	24	7	1
3	0	2	13	901	1	14	1	1	14	10
4	0	1	16	2	904	7	9	10	12	53
5	6	0	3	25	0	810	25	1	11	7
6	8	4	16	5	8	14	905	0	13	1
7	1	0	24	17	1	6	0	931	13	24
8	8	5	41	23	9	19	3	6	875	8
9	1	0	5	8	45	8	0	26	9	884

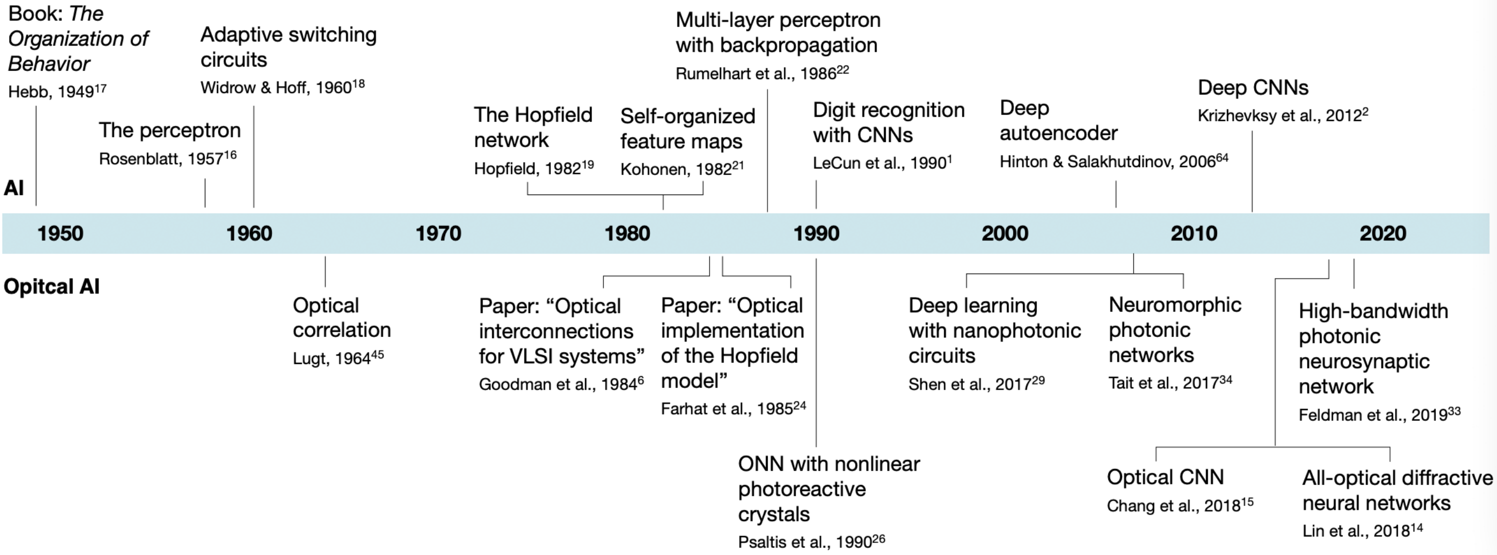
[Lin et al., 2018]

Photonic Integrated Circuit



[Shen et al., 2017]

AI and Related Optical Implementations

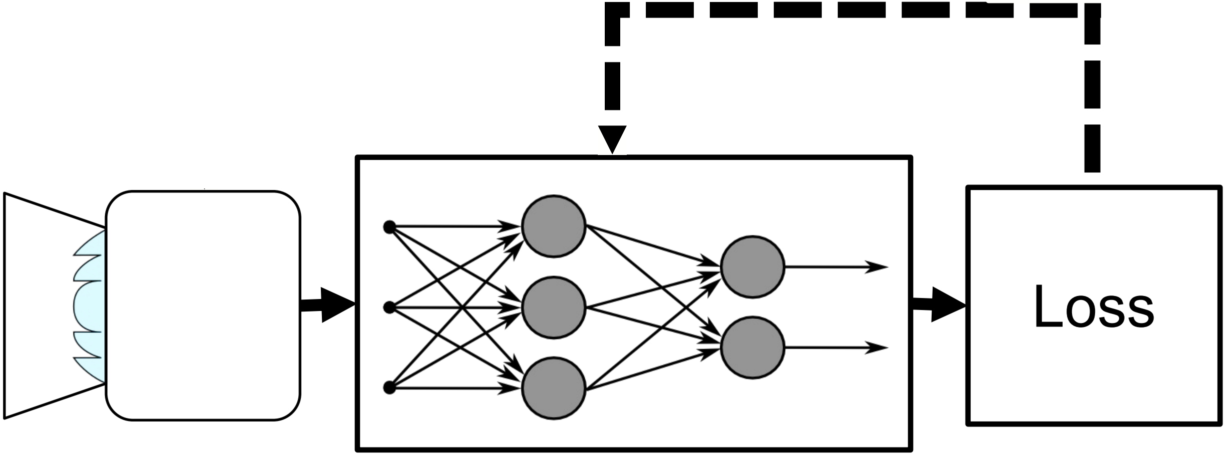


[Wetzstein et al., 2020]

Deep Optics

End-to-end Optimization of Optics and Image Processing

Deep Optics

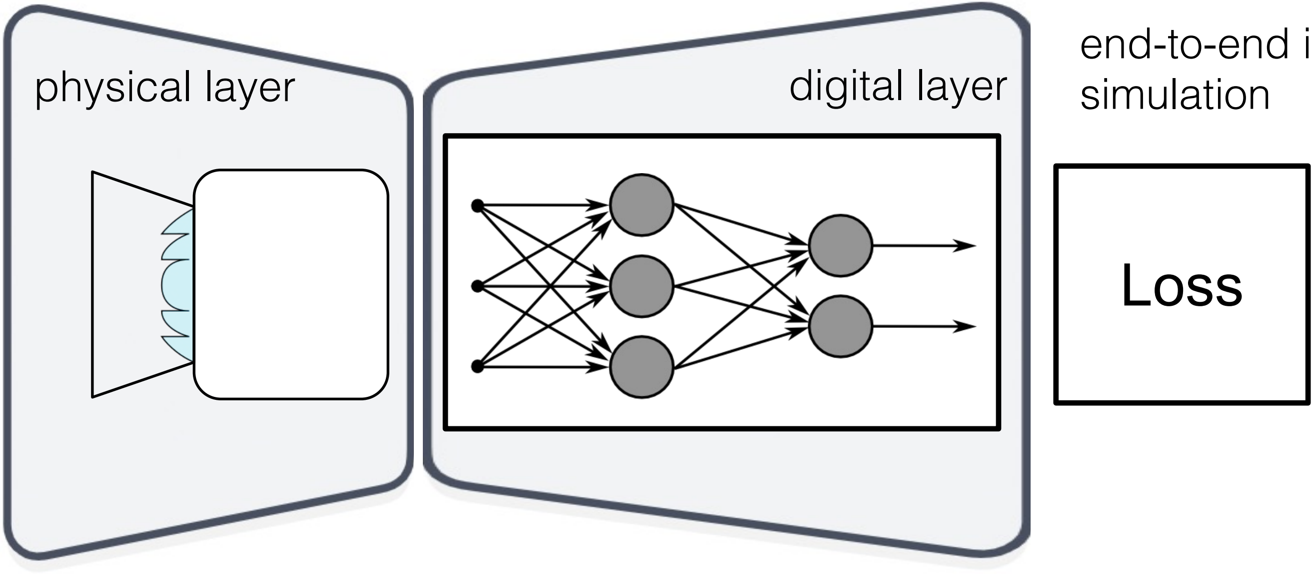


Jointly optimize optics and image processing end-to-end!

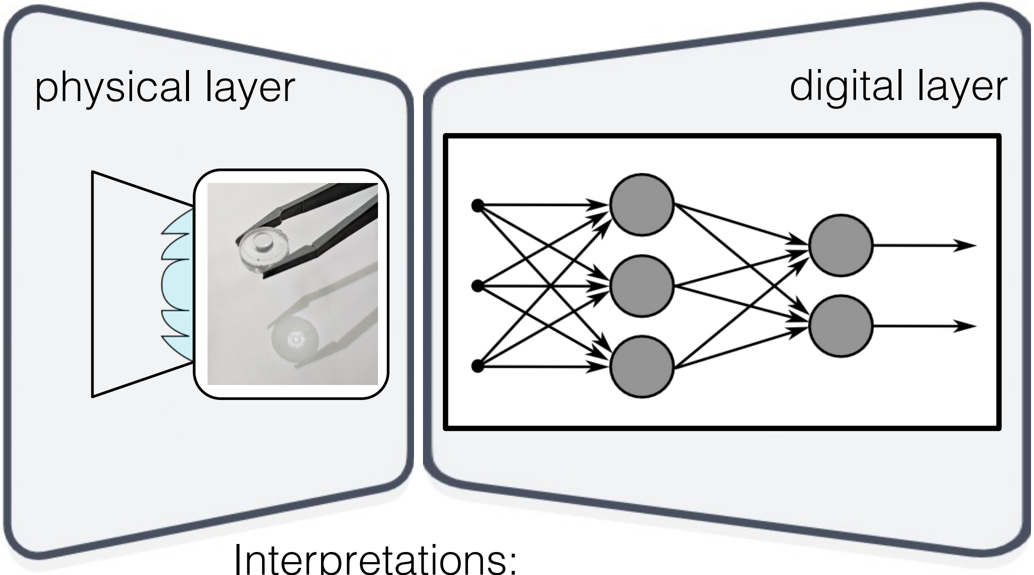
Deep Optics

Training:

end-to-end in simulation



Deep Optics



Inference:

fabricate lens or other physical components, run network

Interpretations:

- Optical encoder, electronic decoder system
- Hybrid optical-electronic neural network

Case Study:

Image Classification in Low Light

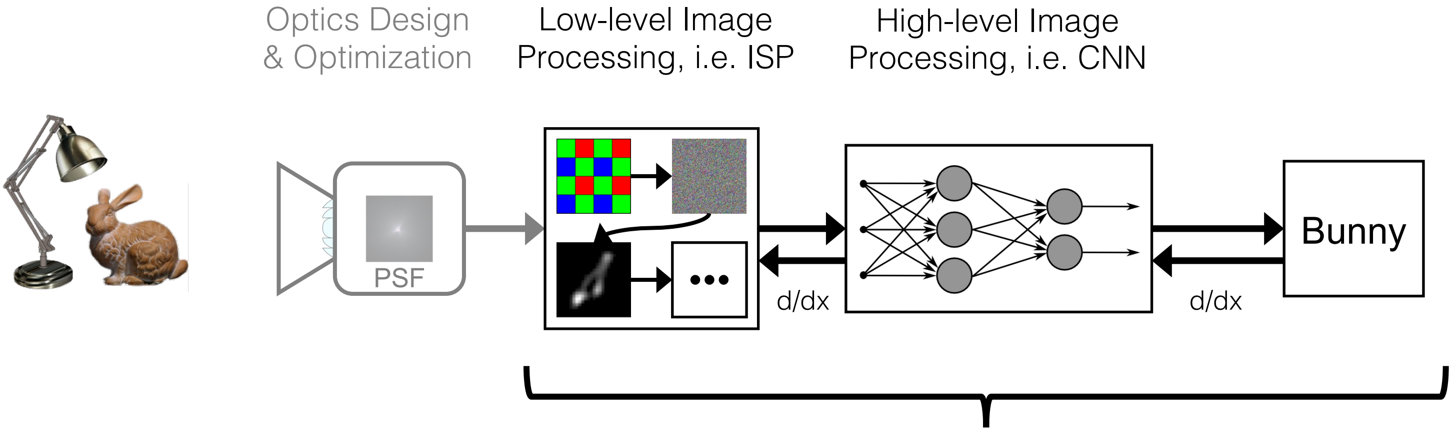
S. Diamond, V. Sitzmann, S. Boyd, G. Wetzstein, F. Heide "**Dirty Pixels: Optimizing Image Classification Architectures for Raw Sensor Data**", arXiv preprint arXiv:1701.06487, 2017

S. Diamond, V. Sitzmann, F. Heide, G. Wetzstein "**Unrolled Optimization with Deep Priors**", arXiv preprint: arXiv:1705:08041, 2018

A classification task

Low-Light
Mobile
Imaging
Scenario

BM3D → Inception-v4 Classification



differentiable pipeline → optimize end-to-end

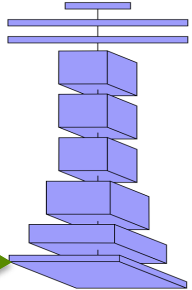
Learning Image Processing

Unrolling Image Optimization

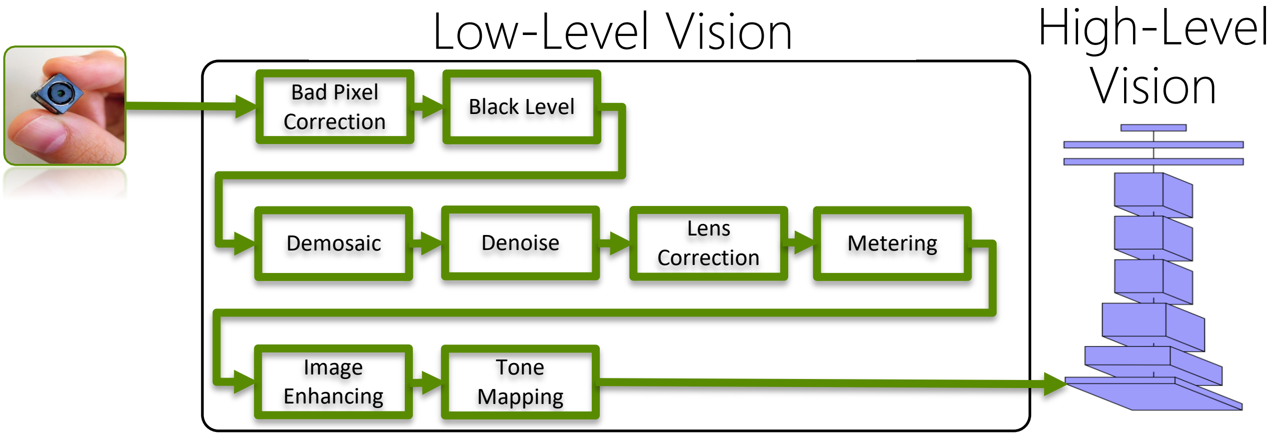
Low-Level Vision

- Physical image formation
- Prior and hyperparameters free
- Differentiable

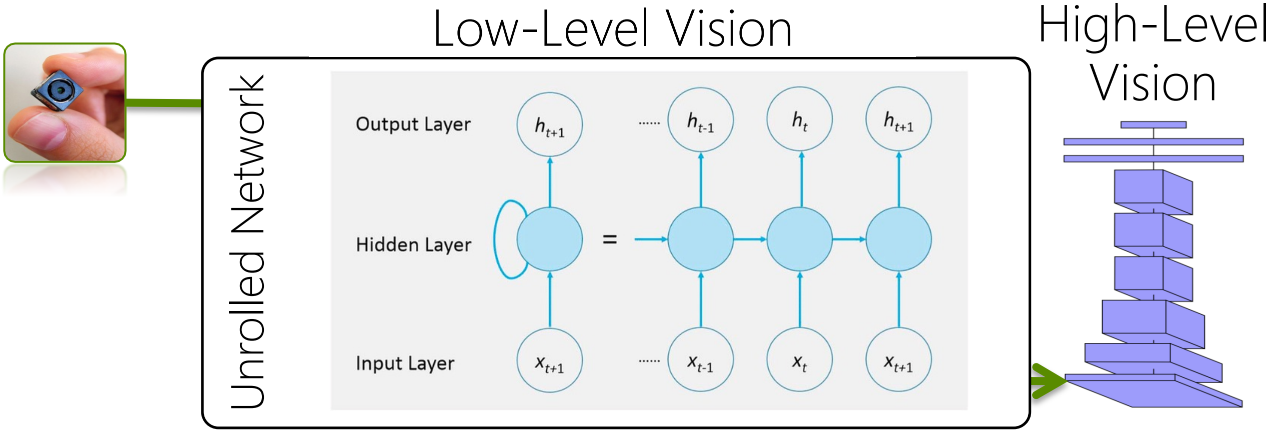
High-Level Vision



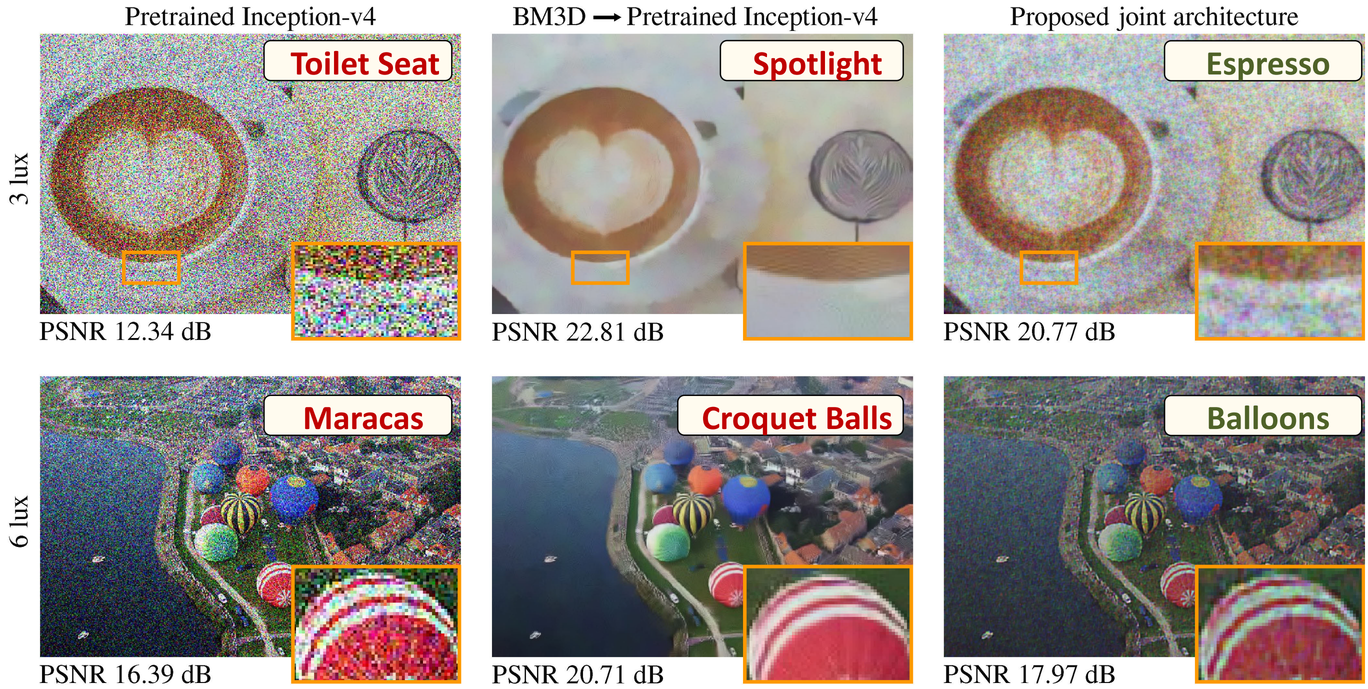
Unrolling Image Optimization



Unrolling Image Optimization



Low-Light Classification

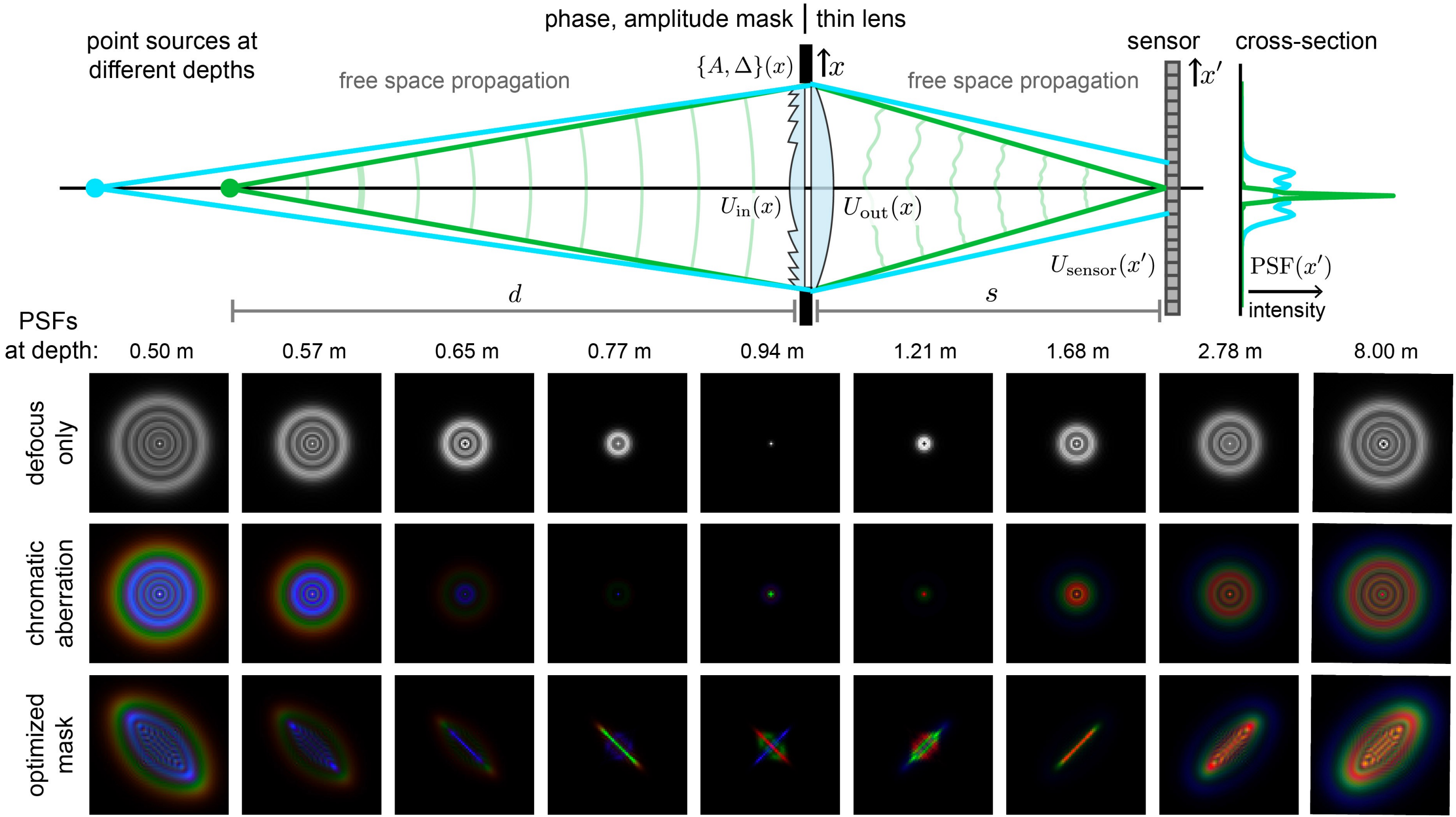


Case Study:

Monocular Depth Estimation

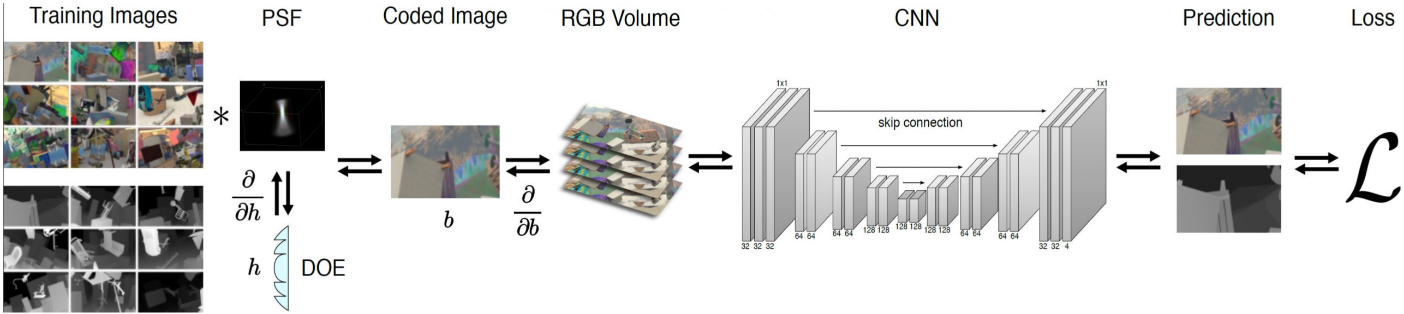
J. Chang, G. Wetzstein "**Deep Optics for Monocular Depth Estimation and 3D Object Detection**", ICCV 2019

H. Ikoma, C. Nguyen, Y. Peng, G. Wetzstein "**Depth from Defocus with Learned Optics for Imaging and Occlusion-aware Depth Estimation**", ICCP 2021



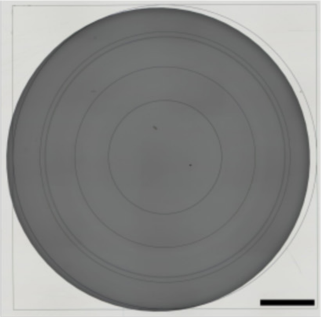
phase, amplitude mask | thin lens
point sources at
different depths
free space propagation
$\{A, \Delta\}(x)$
x
free space propagation
sensor
x'
cross-section
$U_{\text{in}}(x)$
$U_{\text{out}}(x)$
$U_{\text{sensor}}(x')$
$\text{PSF}(x')$
intensity
d
s
PSFs
at depth:
0.50 m
0.57 m
0.65 m
0.77 m
0.94 m
1.21 m
1.68 m
2.78 m
8.00 m
defocus
only
chromatic
aberration
optimized
mask

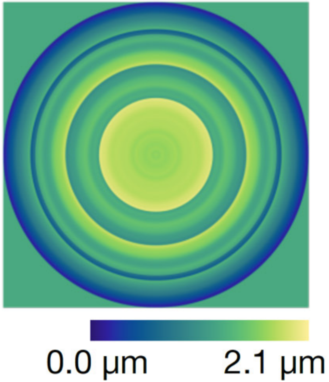
Pipeline



Ikoma et al., ICCP 2021

Prototype

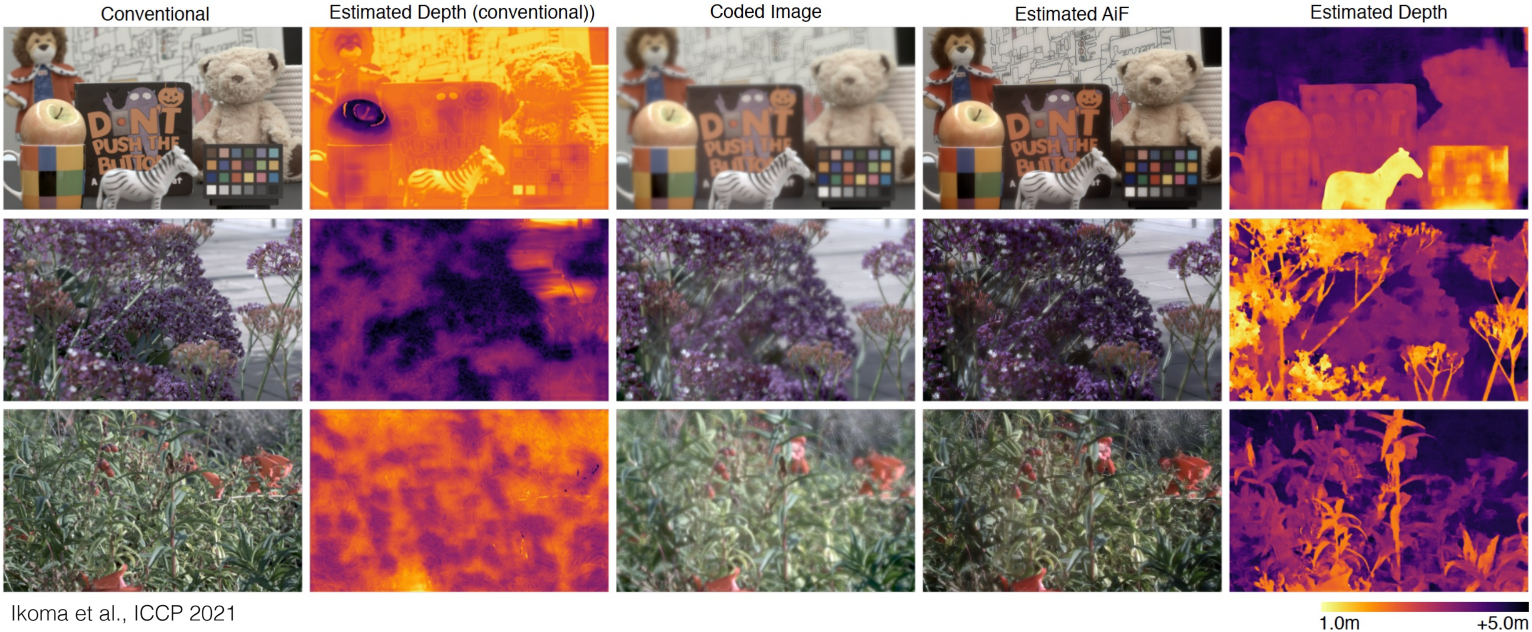




Ikoma et al., ICCP 2021

Results

- Estimate RGB image and depth map from a single optically coded image



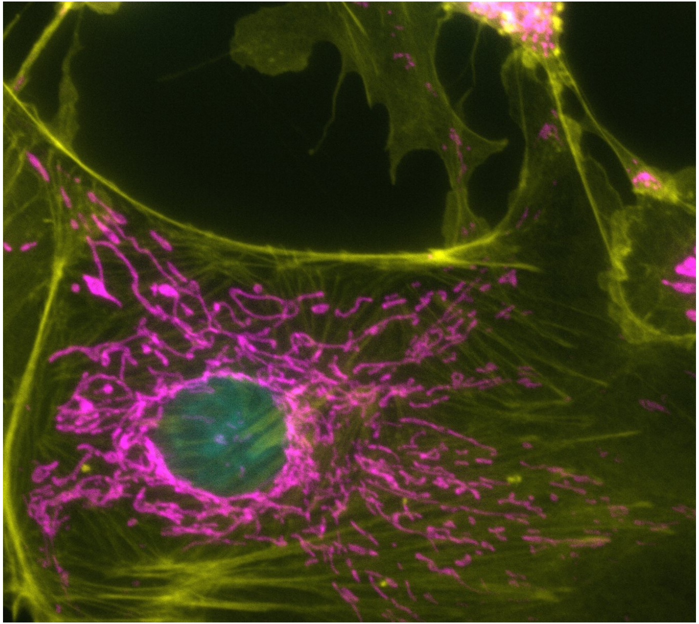
Ikoma et al., ICCP 2021

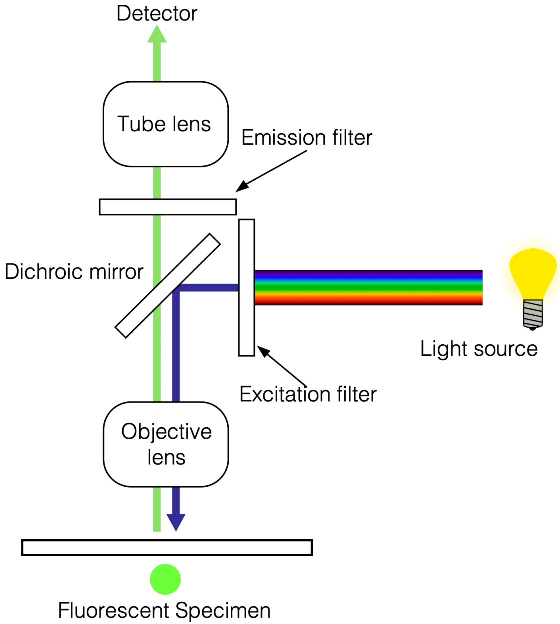
Case Study:

3D Localization Microscopy

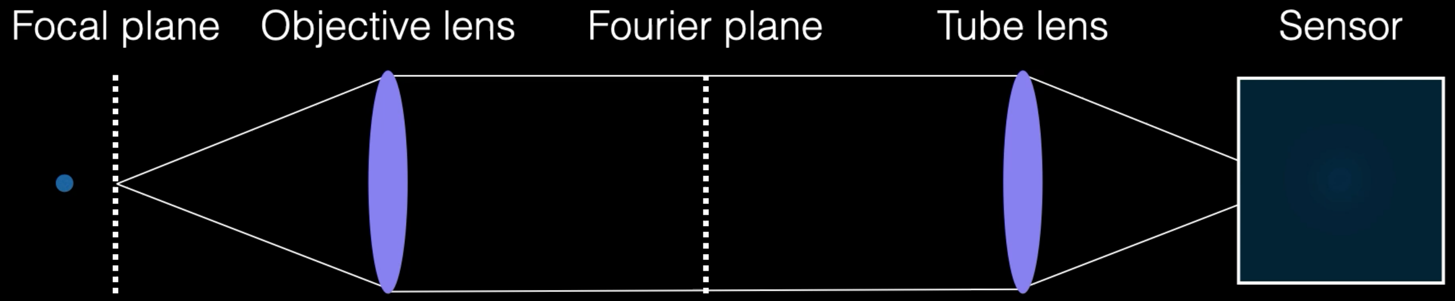
H. Ikoma, T. Kudo, Y. Peng, M. Broxton, **“Deep learning multi-shot 3D localization microscopy using hybrid optical-electronic computing”**, in submission

Fluorescence Microscopy

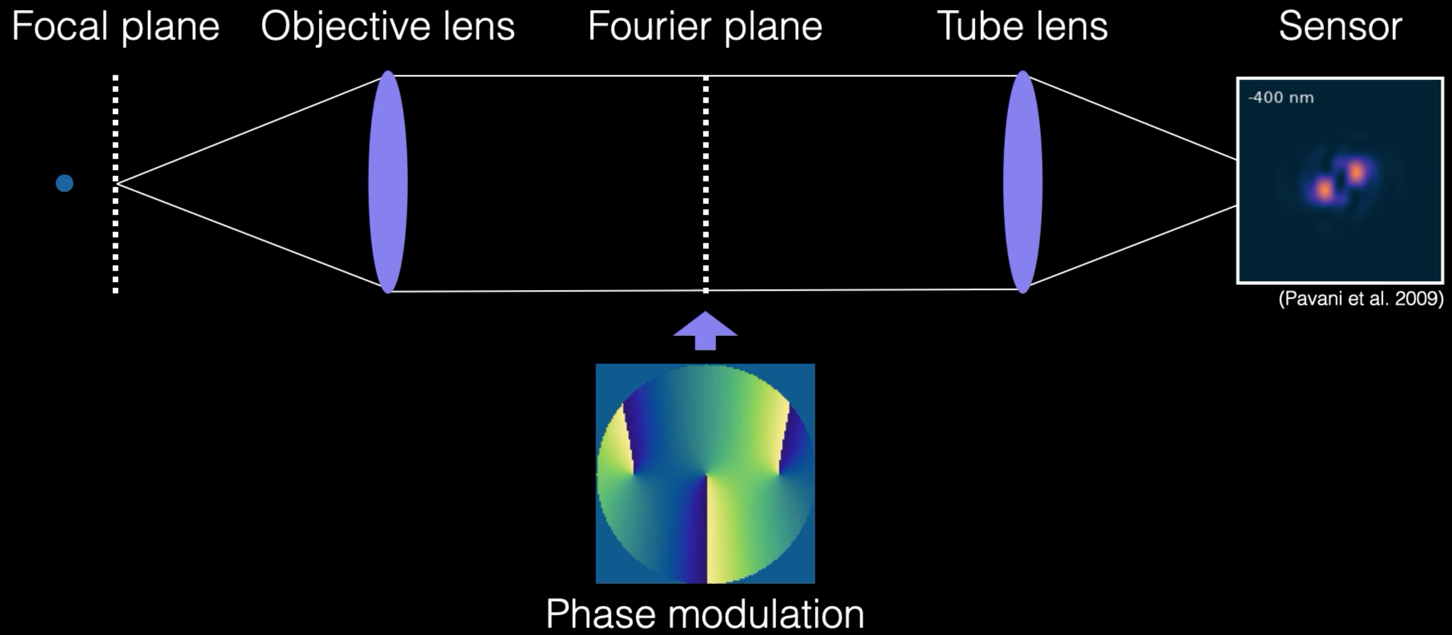




PSF of a widefield microscope



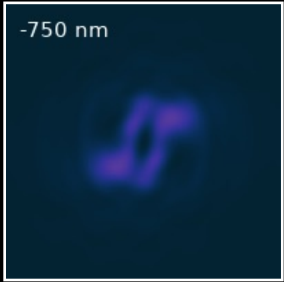
PSF engineering - Double Helix PSF



Other PSFs for Single Molecule Localization

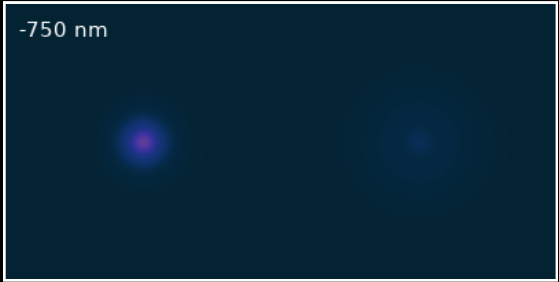
Astigmatism

Double helix



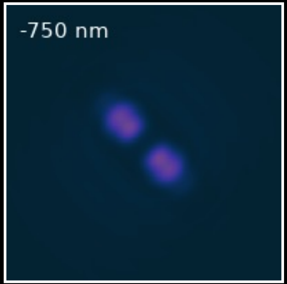
(Pavani et al. 2009)

Biplane



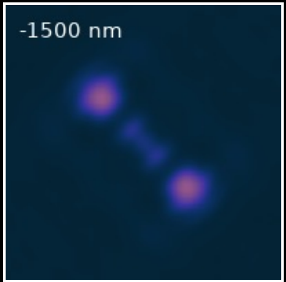
(Juette et al. 2008)

Saddle-point

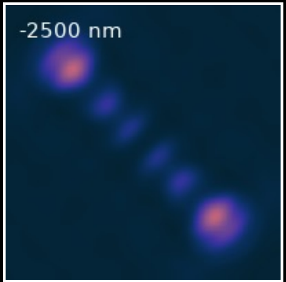


(Shechtman et al. 2014)

Tetrapod (3μm)

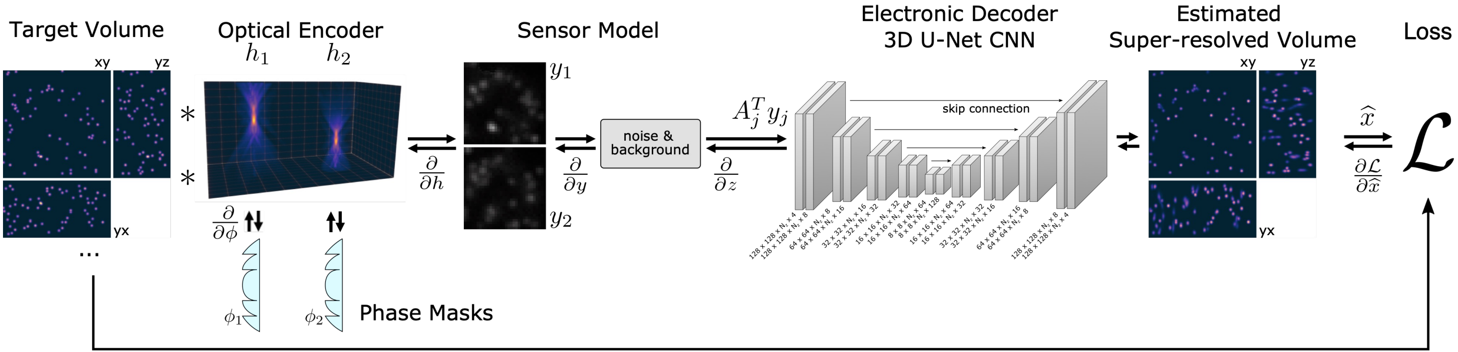


Tetrapod (5μm)

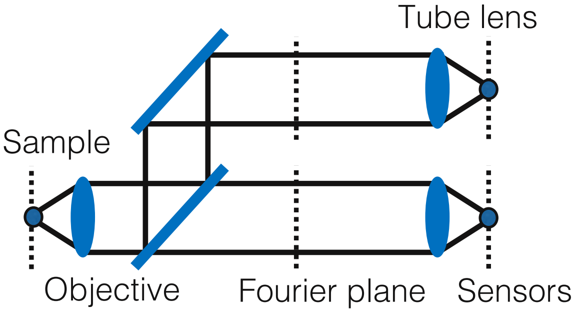


(Shechtman et al. 2015)

Deep Learning–based PSF Engineering

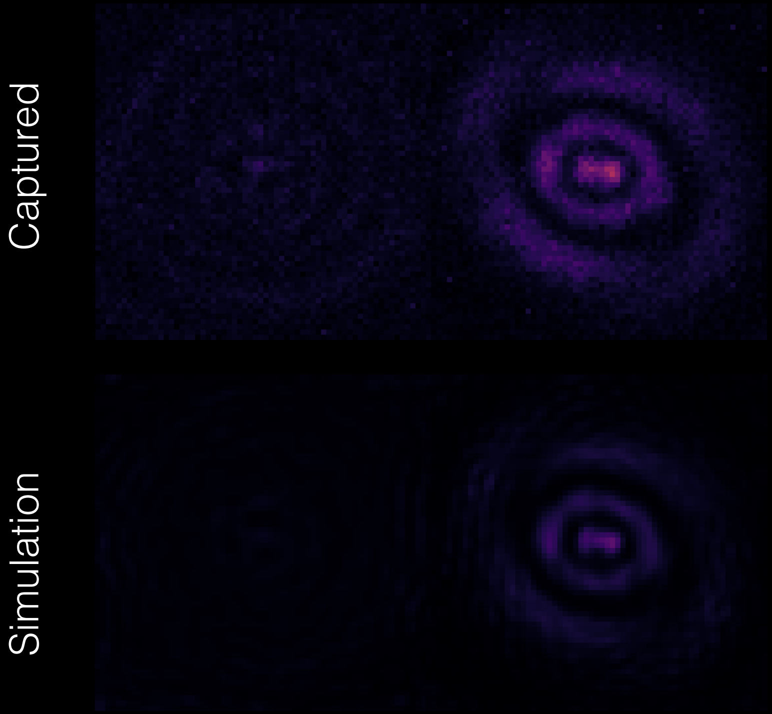


Multi-path imaging system



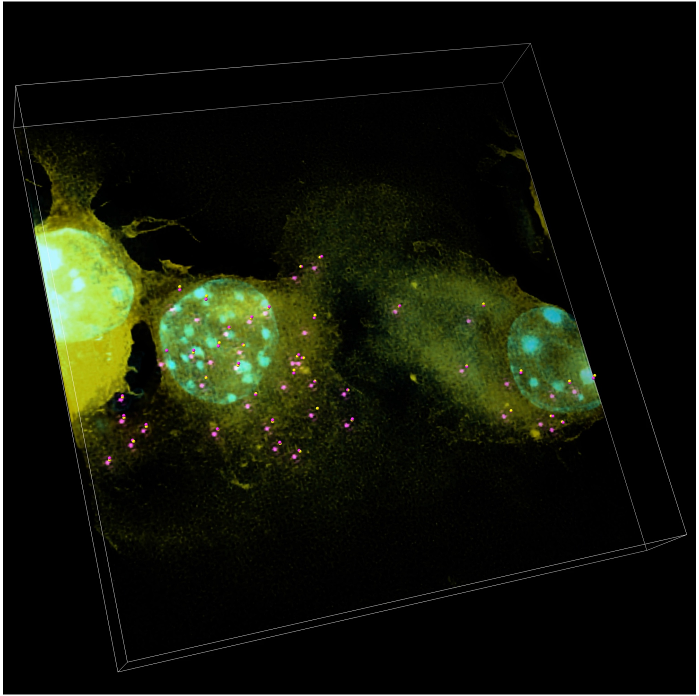
Ikoma et al

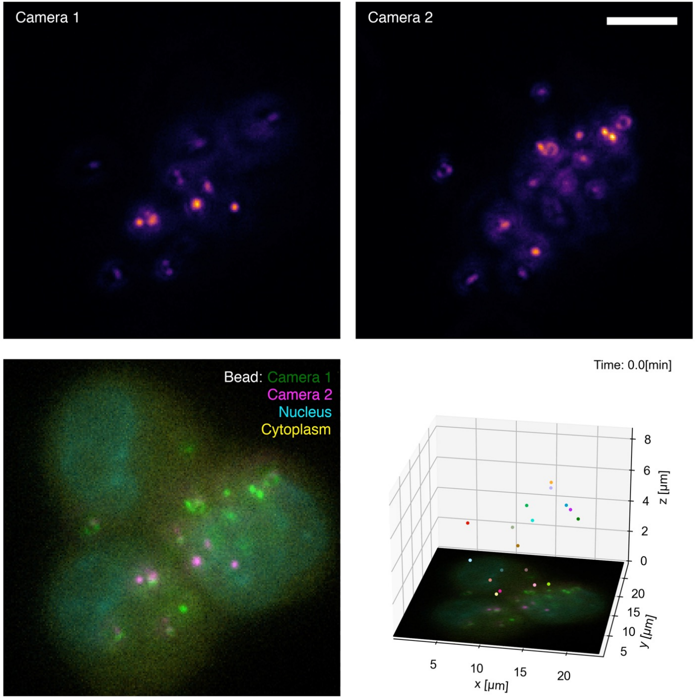
Optimized Two-shot 3D PSFs



Ikoma et al.

Preliminary Results





Fixed cells with beads

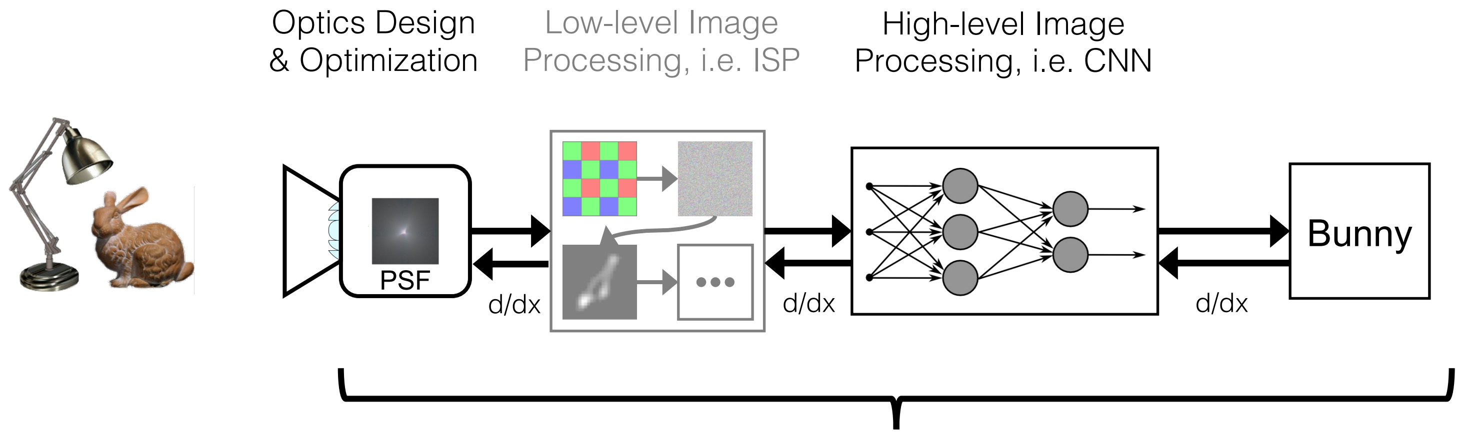
Time lapse of live cells

Ikoma et al.

Case Study:

Hybrid Optical-Electronic Computing

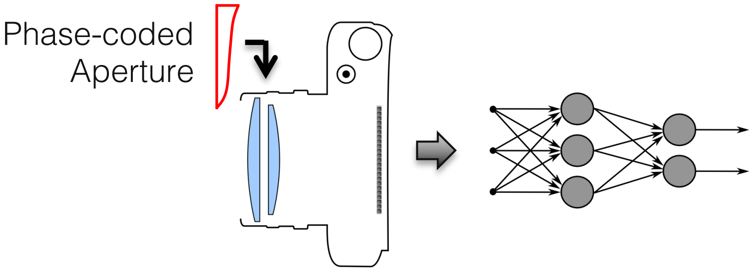
J. Chang, V. Sitzmann, X. Dun, W. Heidrich, G. Wetzstein "**Hybrid optical-electronic convolutional neural networks with optimized diffractive optics for image classification**", Scientific Reports, 2018

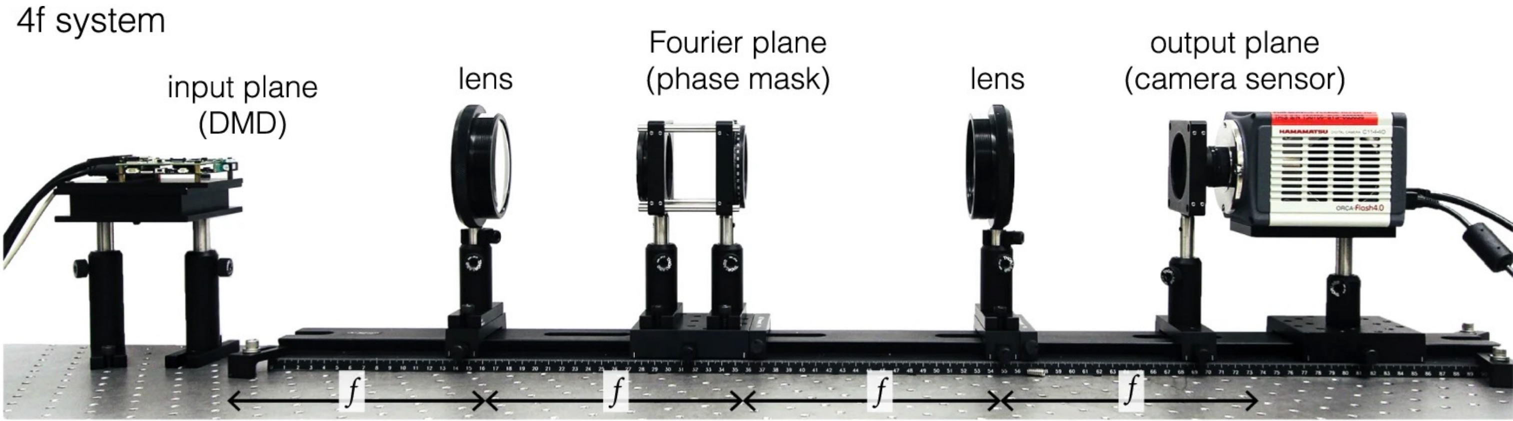


differentiable pipeline → optimize end-to-end

Learning Optics & CNN

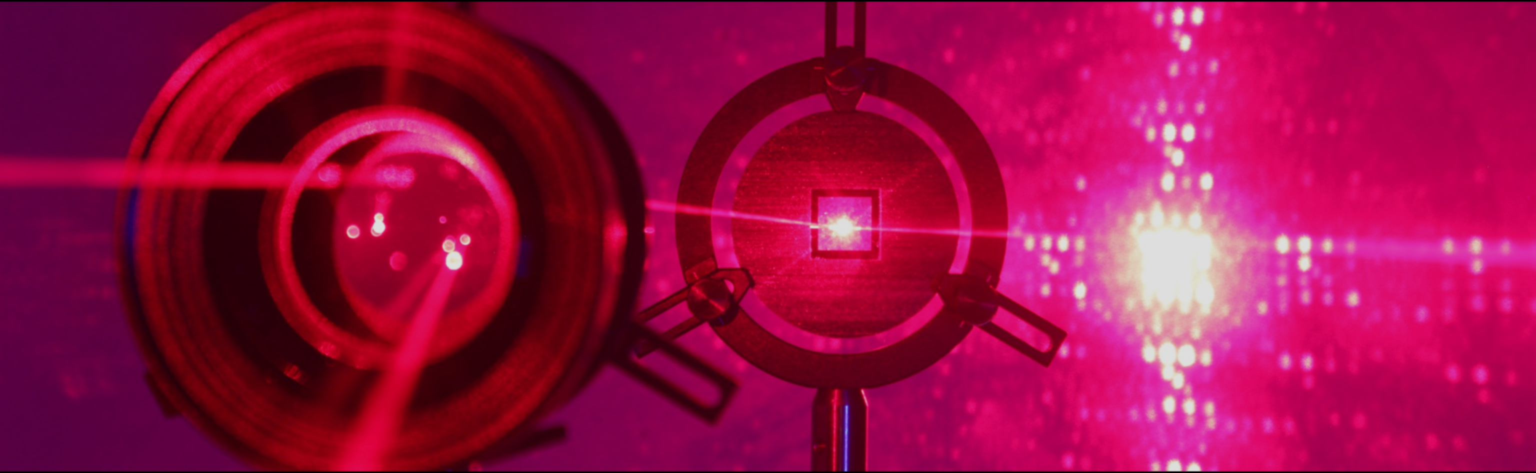
Hybrid Optical-Electronic CNNs





Chang et al., Scientific Reports 2018

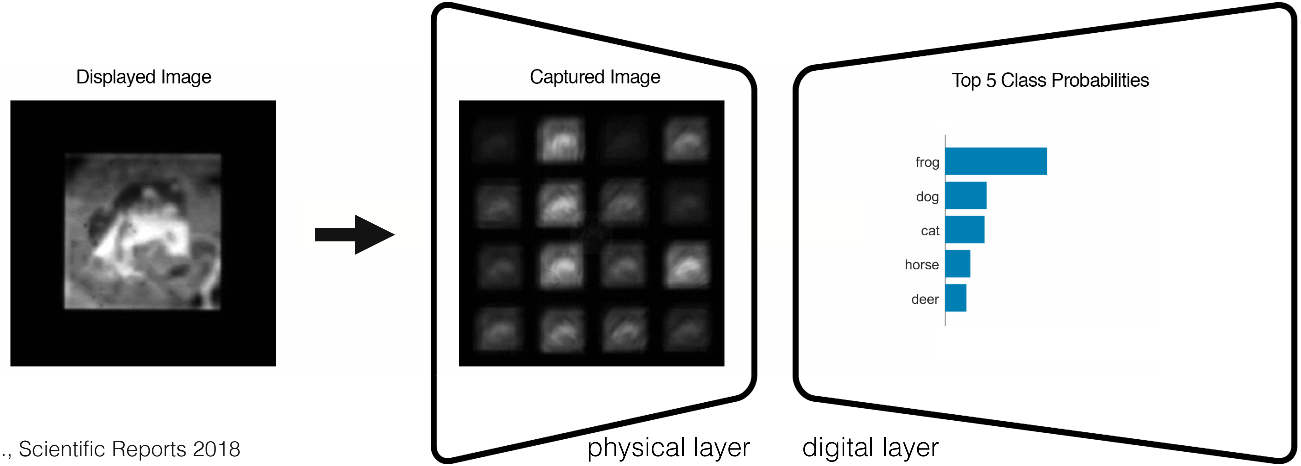
Chang et al., Scientific Reports 2018



Hybrid Optical-Electronic CNNs

current results:

- 2x classification accuracy for same power
- half power for same classification accuracy



Chang et al., Scientific Reports 2018

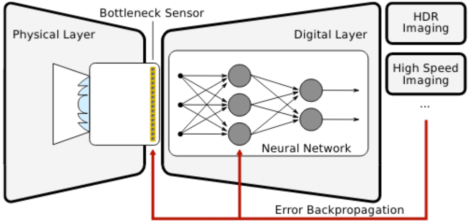
Case Study:

Neural Sensors

J. Martel, L. Muller, S. Carey, P. Dudek, G. Wetzstein "**Neural Sensors: Optimizing Pixel Exposures for HDR Imaging and Video Compressive Sensing with Programmable Sensors**", IEEE TPAMI (Proc. ICCP) 2020

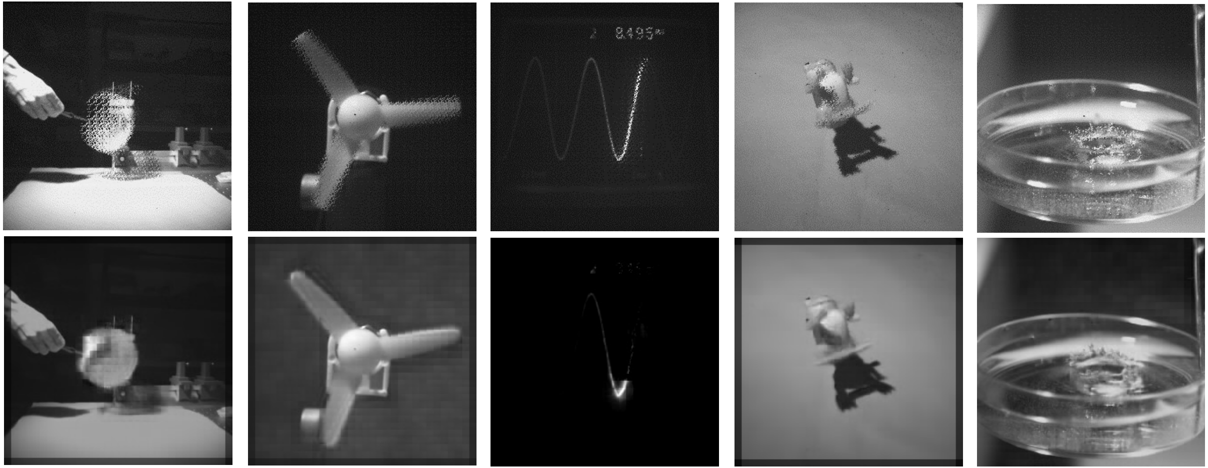
Y. Li, M. Qi, R. Gulve, M. Wei, R. Genov, K. Kutulakos, W. Heidrich "**End-to-End Video Compressive Sensing Using Anderson-Accelerated Unrolled Networks**", ICCP 2020

Neural Sensors



Coded Measurements

Reconstructions



Martel et al., ICCP 2020

Other Examples

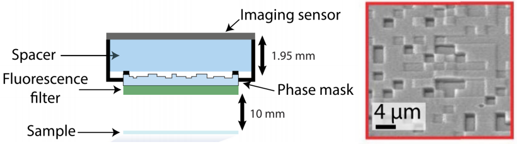
HDR Imaging

Metzler et al. CVPR 2020

Sun et al. CVPR 2020

EDOF Imaging

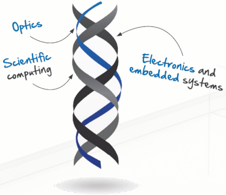
Sitzmann et al. SIGGRAPH 2018



Flat / Lensless Cameras

Boominathan et al. TPAMI/ICCP 2020

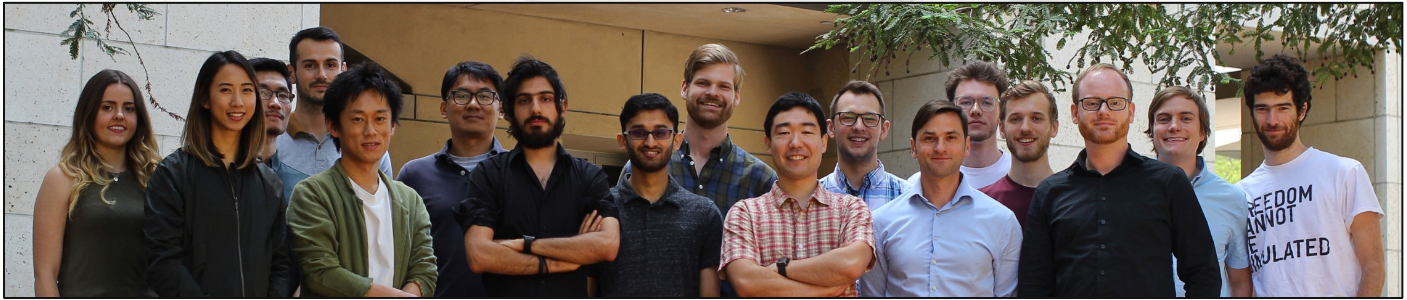
Gordon Wetzstein
stanford.edu/~gordonwz



Computational Imaging Lab
Stanford University EE & CS

G. Wetzstein, A. Ozcan, S. Gigan, S. Fan, D. Englund, M. Soljacic, C. Denz, D. Miller, D. Psaltis, “**Inference in artificial intelligence with deep optics and photonics**”, Nature (review article), 2020

computationalimaging.org



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References and Further Reading

Wave Optics:

- J. Goodman, “Introduction to Fourier Optics”, Oxford University Press, 2012
- A. Lugt, “Signal detection by complex spatial filtering”, IEEE Trans. Information Theory, 1964
- J. Chang, V. Sitzmann, X. Dun, W. Heidrich, G. Wetzstein, “Hybrid optical-electronic convolutional neural networks with optimized diffractive optics for image classification”, Scientific Reports 2018
- G. Wetzstein, A. Ozcan, S. Gigan, S. Fan, D. Englund, M. Soljačić, C. Denz, D. Miller, D. Psaltis, “Inference in artificial intelligence with deep optics and photonics”, Nature (review paper), 2020
- Y. Shen, N. Harris, S. Skirlo, M. Prabhu, T. Baehr-Jones, M. Hochberg, X. Sun, S. Zhao, H. Larochelle, D. Englund, M. Soljačić, “Deep learning with coherent nanophotonic circuits”, Nature Photonics, 2017
- X. Lin, Y. Rivenson, N. Yardimci, M. Veli, Y. Luo, M. Jarrahi, A. Ozcan, “All-optical machine learning using diffractive deep neural networks”, Science, 2018
- Z. Zhang, M. Levoy, “Wigner distributions and how they relate to the light field”, ICCP 2009

Deep Optics:

See individual slides