

Phase Retrieval and Computer-generated Holography

EE367/CS448I: Computational Imaging

stanford.edu/class/ee367

Lecture 15

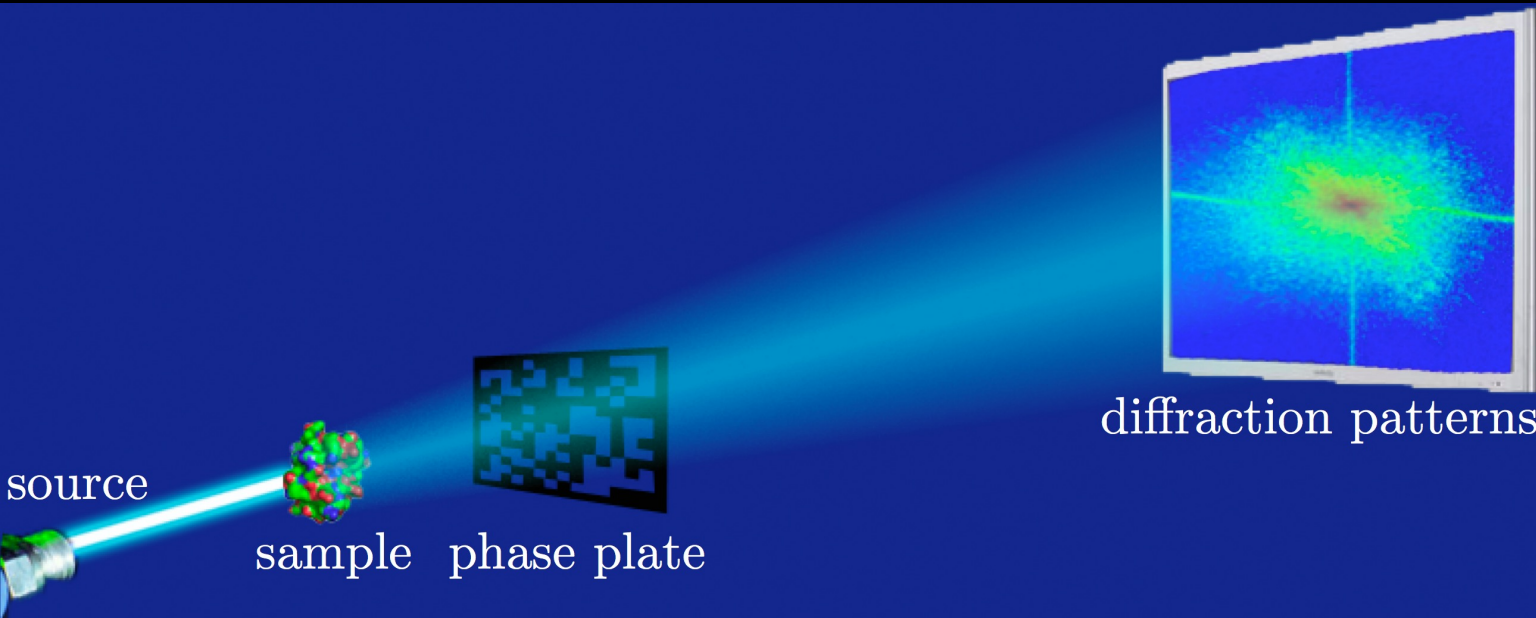
Gordon Wetzstein
Stanford University



Brief Recap of (some) Wave Optics

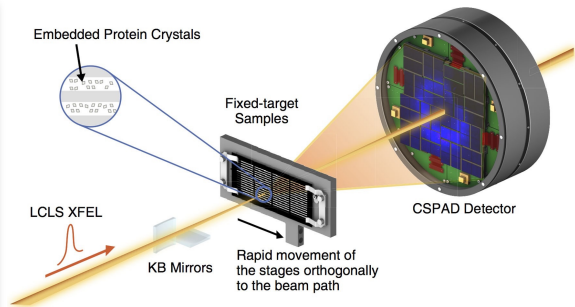
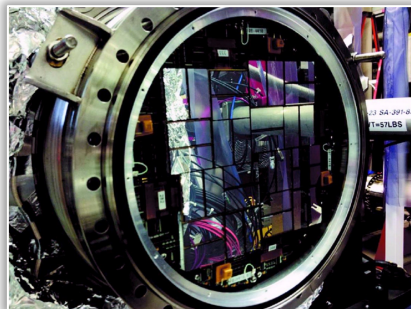
- Free-space wave propagation is often modeled by the Fourier transform of the field, i.e., for large distances or with lenses
- Cannot measure complex field, only intensity (i.e., amplitude squared)

Phase Retrieval



Phase Retrieval @ SLAC

x-ray crystallography or single-particle imaging (SPI)



Applications of Phase Retrieval

- Crystallography
- Transmission electron microscopy
- Astronomy
- Coherent diffractive imaging
- Fourier ptychography
- Lensless imaging
- Computer-generated holography
- ...

Phase Retrieval

- Objective function: $\text{minimize}_x J(x) = \frac{1}{2} \| |Fx| - b \|_2^2$
- F is discrete Fourier transform matrix, b are amplitude-only measurements, x is complex-valued unknown vector
- Subgradient of objective: $F^H \left((|Fx| - b) \circ \frac{Fx}{|Fx|} \right) \in \nabla J(x)$
- Absolute value not differentiable, but can work with subgradients

Phase Retrieval

- (Sub)gradient descent:

$$\begin{aligned}x^{(k+1)} &= x^{(k)} - \alpha \nabla J(x) \\ &= x^{(k)} - \alpha F^H F x^{(k)} + \alpha F^H \left(b \circ \frac{F x^{(k)}}{|F x^{(k)}|} \right) \\ &\stackrel{\alpha=1}{\approx} F^H \left(b \circ \frac{F x^{(k)}}{|F x^{(k)}|} \right)\end{aligned}$$

- Interesting: $b \circ \frac{a e^{i\phi}}{|a e^{i\phi}|} = b e^{i\phi}$

Phase Retrieval

- Generalized Gerchberg-Saxton or Error Reduction (ER) algorithm [GS 1972, Fienup 1982]:

$$x^{(k+1)} = \Pi_{\mathcal{C}} \left(F^H \left(b \circ \frac{F x^{(k)}}{|F x^{(k)}|} \right) \right)$$

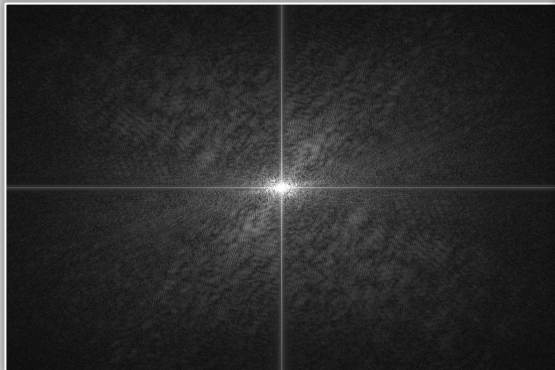
- Projection on feasible set $\Pi_{\mathcal{C}}$ enforces additional constraints, such as nonnegativity of x or limited support, via projected (sub)gradient descent
- Approach is 40 years old, but should be a great starting point

Phase Retrieval - Example

Ground Truth Image



Simulated measurements



Reconstructed Image

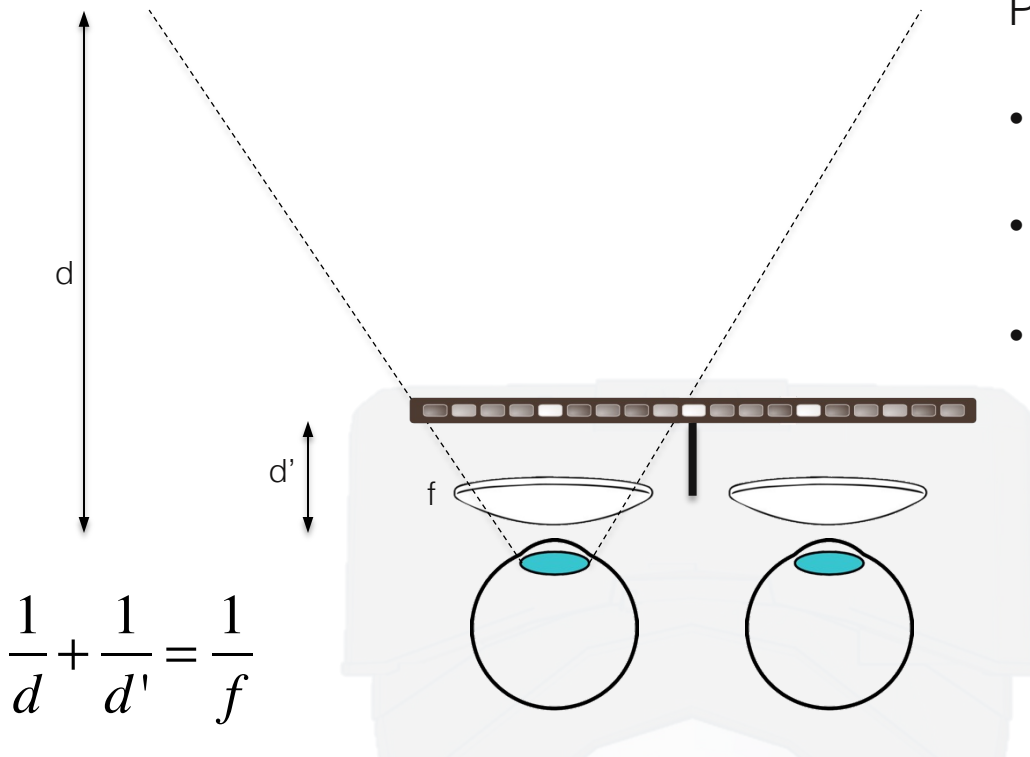
Holographic Displays



Sir Charles Wheatstone, 1838



Virtual Image



Problems:

- fixed focal plane
- no focus cues ☹️
- vergence-accommodation conflict (nausea)

Computational Near-eye Displays with Focus Cues

Gaze-contingent Varifocal Displays



Shiwa et al. 1996; Liu et al. 2008;
Konrad et al. 2016; Padmanaban et al. 2016, 2017; ...

Multiplane Displays



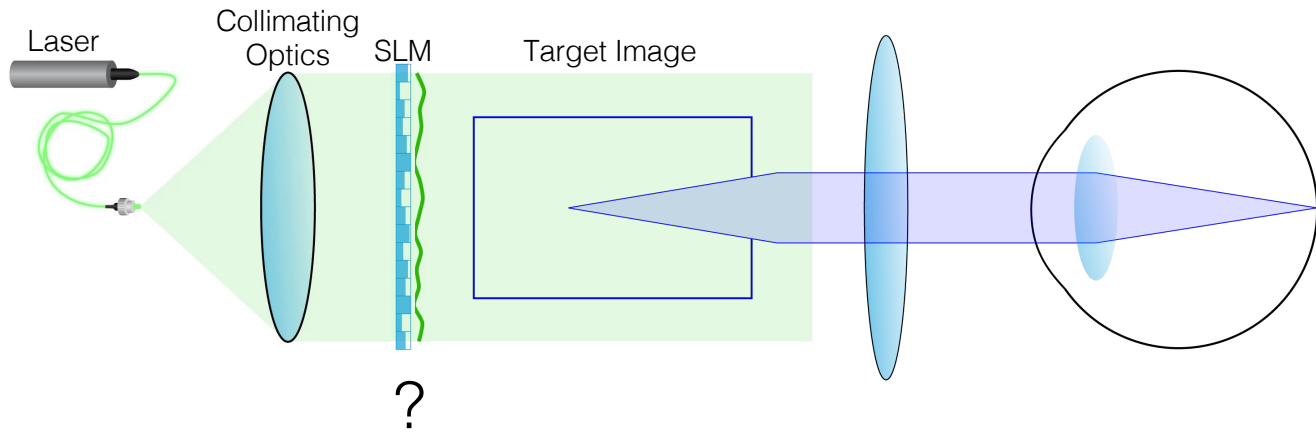
Rolland et al. 2000; Akeley et al. 2004
Liu et al. 2008; Love et al. 2009; ...

Near-eye Light Field Displays

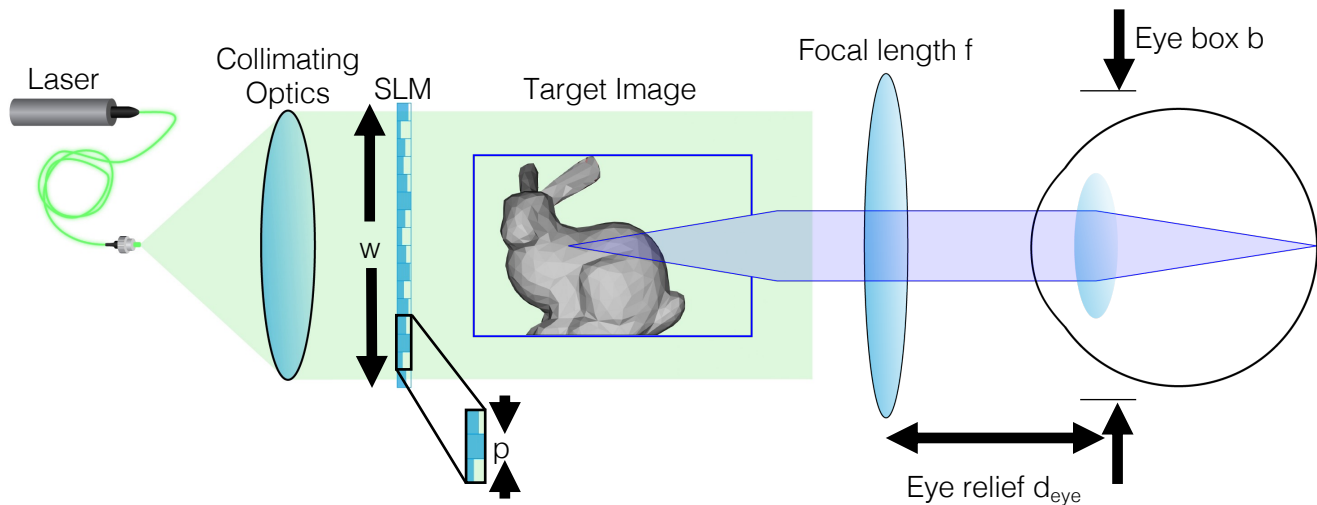


Lanman and Luebke 2013;
Hua and Javidi 2014; Huang et al. 2015

Holographic Near-eye Displays



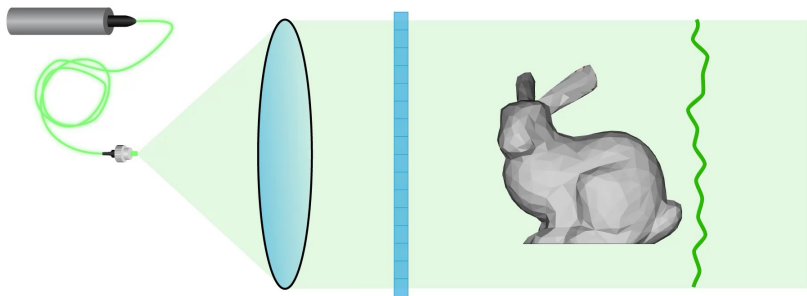
Holographic Near-eye Displays



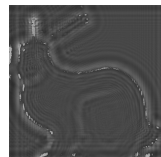
$$\text{Field of view } FOV \approx 2 \tan^{-1} \left(\frac{w}{2f} \right)$$

$$\text{Eye box } b \approx d_{eye} \frac{\lambda}{p}$$

Computer-generated Holography: Direct Methods



SLM phase

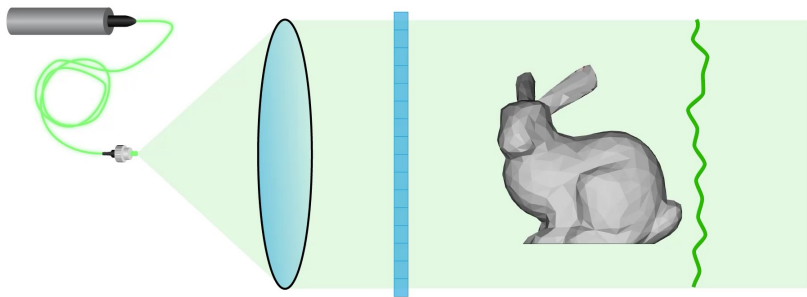


propagate
backward

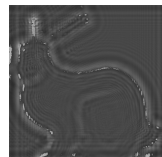
Target Image



Computer-generated Holography: Direct Methods



SLM phase



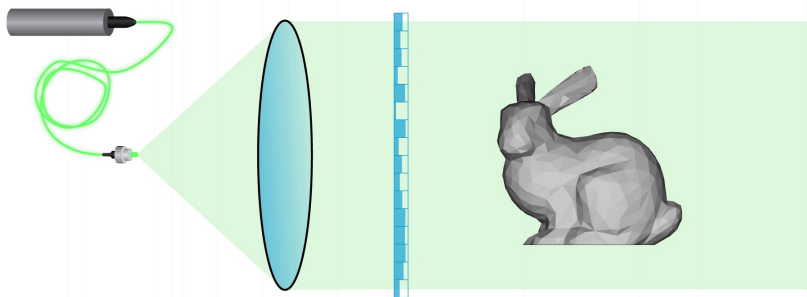
Target Image



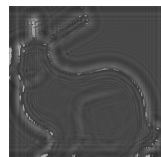
propagate
backward

1. Only target intensity is provided, need to “make up” some target phase (e.g., 0)
2. Free-space propagation from target to SLM plane
3. Propagated field at SLM plane is complex, but SLM can only address phase → need phase encoding

Computer-generated Holography: Direct Methods



SLM phase



Target Image



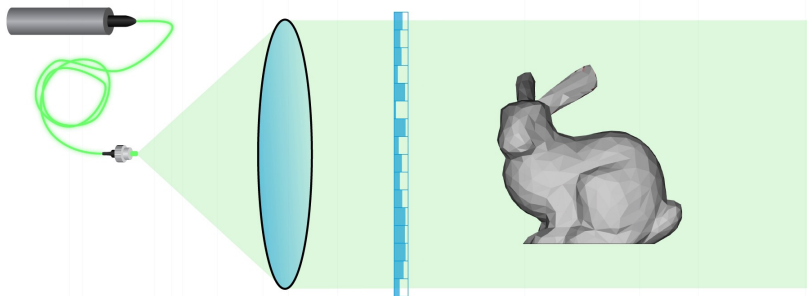
propagate
backward

Free-space propagation:

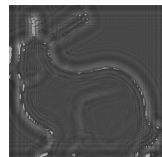
$$u_{slm}(x, y) = \mathcal{F}^{-1} \left\{ \mathcal{F} \left\{ a(x, y) e^{i\phi(x, y)} \right\} \mathcal{H}(f_x, f_y, z) \right\}$$

$$\mathcal{H}(f_x, f_y) = \begin{cases} e^{-i \frac{2\pi}{\lambda} \sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2} z} & \text{if } \sqrt{f_x^2 + f_y^2} < \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases}$$

Computer-generated Holography: Direct Methods



SLM phase



Target Image



propagate
backward

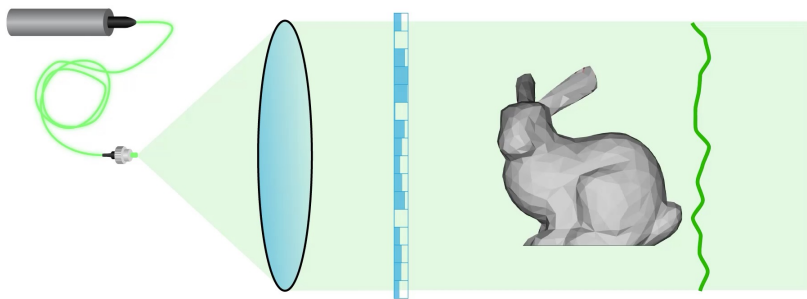
Double phase-amplitude coding:

$$u_{slm}(x, y) = a(x, y) e^{i\phi(x, y)} = 0.5 \left(e^{i\phi_1(x, y)} + e^{i\phi_2(x, y)} \right)$$

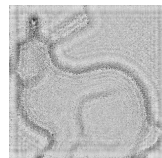
$$\phi_1(x, y) = \phi(x, y) - \cos^{-1}(a(x, y))$$

$$\phi_2(x, y) = \phi(x, y) + \cos^{-1}(a(x, y))$$

Computer-generated Holography: Iterative Methods



SLM phase



Target Image



propagate
backward

propagate
forward

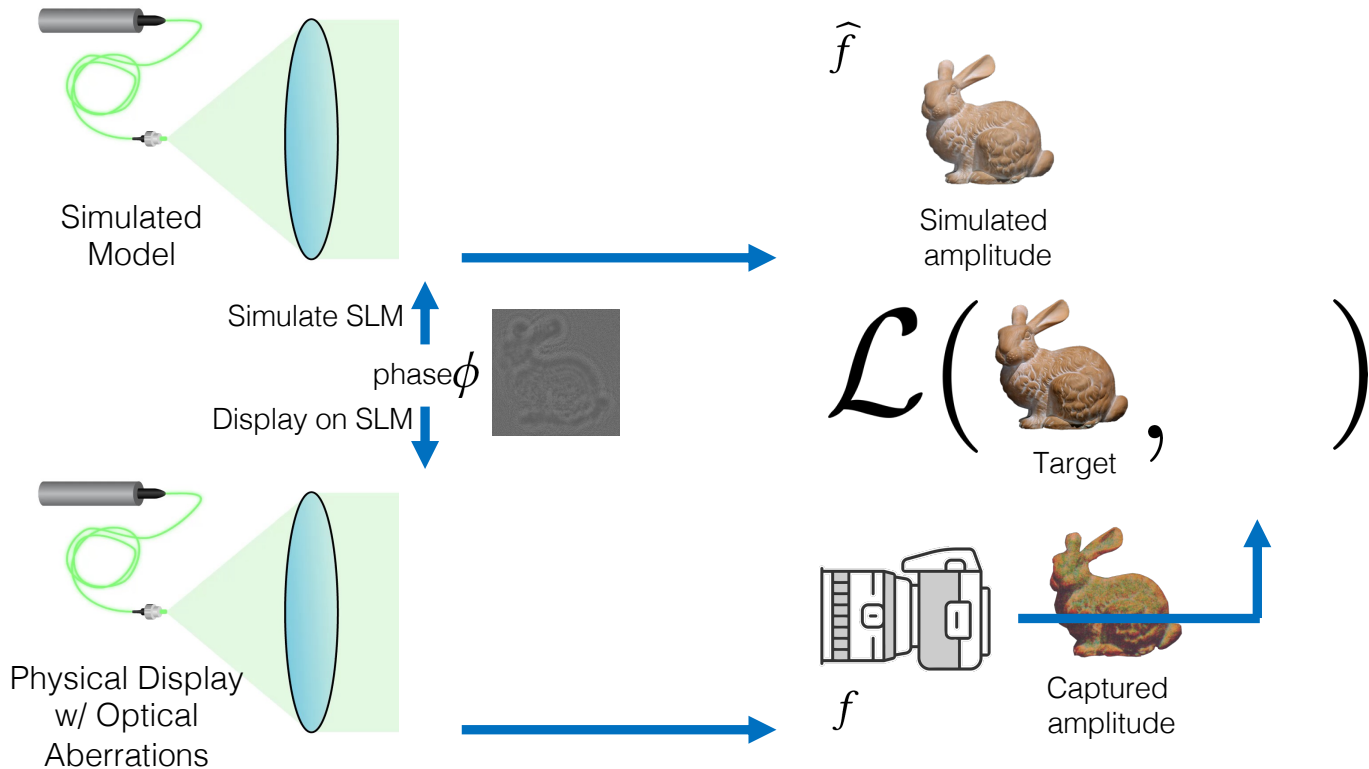
$$\text{minimize}_{\phi} \mathcal{L} \left(\left| \hat{f}(\phi) \right|, \mathbf{a}_{\text{target}} \right)$$

\hat{f} Free-space propagation model

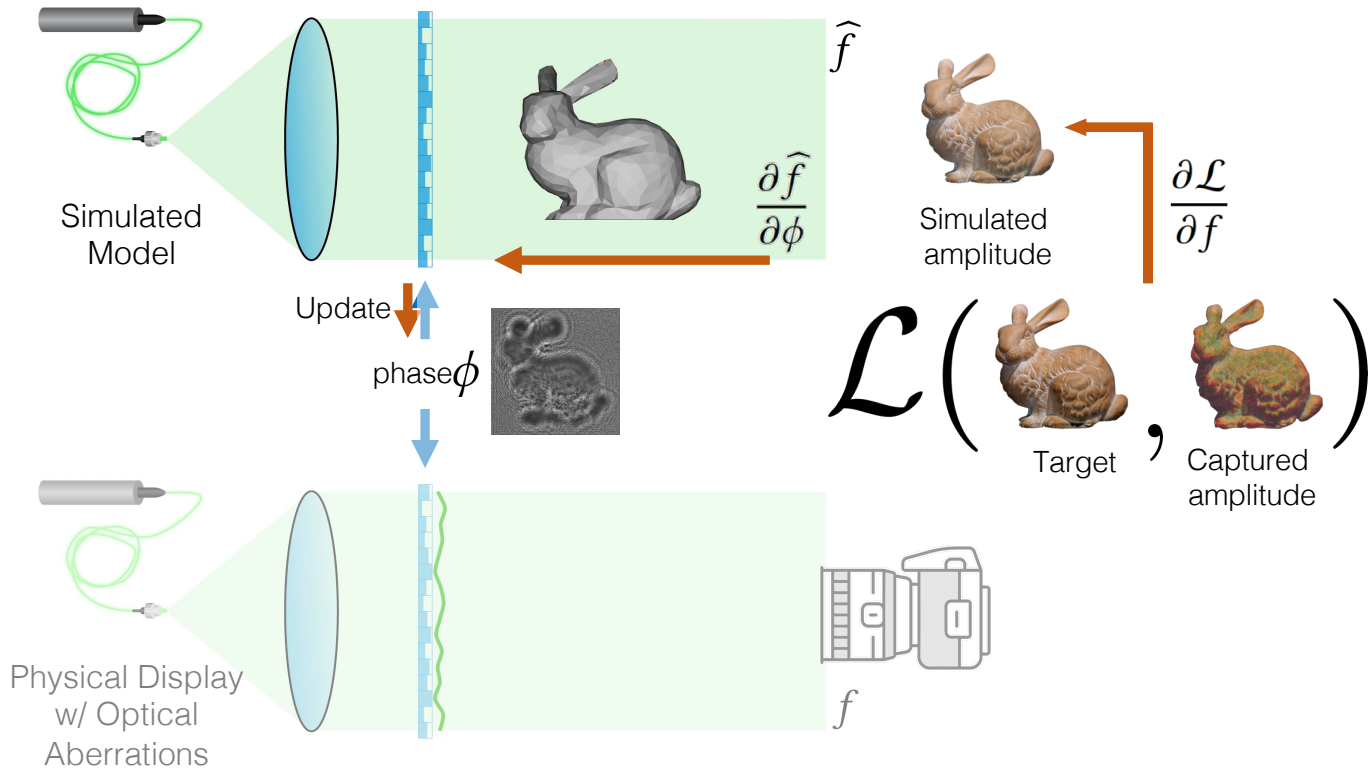
f Unknown physical propagation, $\hat{f} \neq f$

$$\text{Iterations: } \phi^{(k)} \leftarrow \phi^{(k-1)} - \alpha \left(\frac{\partial \mathcal{L}}{\partial \phi} \right)^T \mathcal{L} \left(\left| \hat{f}(\phi^{(k-1)}) \right|, \mathbf{a}_{\text{target}} \right)$$

Camera-in-the-loop (CITL) Optimization

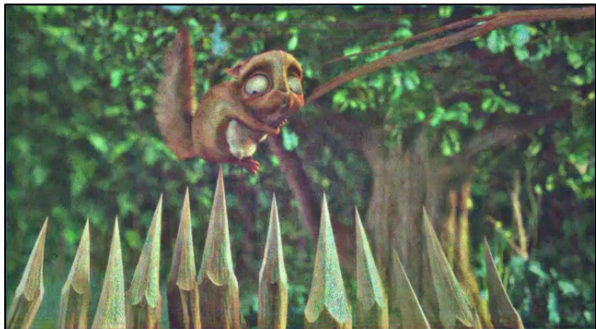


Camera-in-the-loop (CITL) Optimization



Camera-in-the-loop (CITL) Hologram Optimization

SGD with the ASM Model

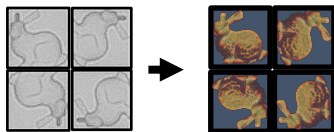
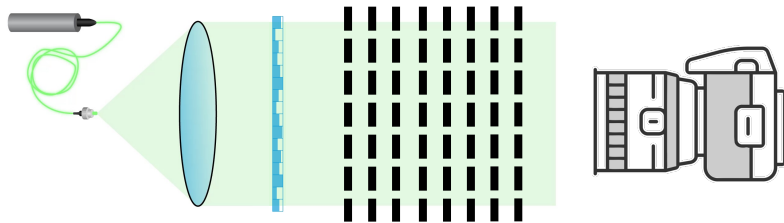


Proposed CITL CGH

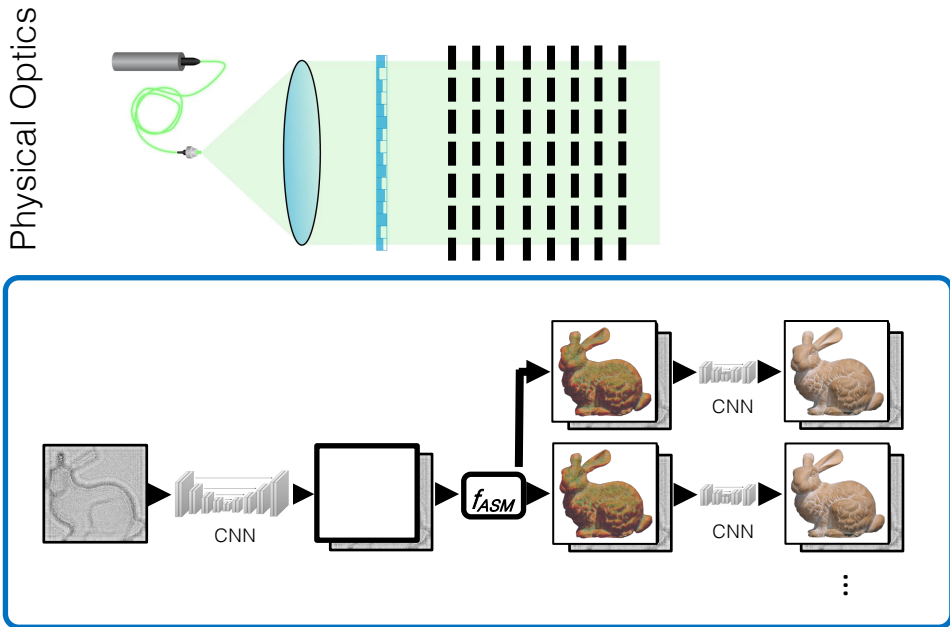


Neural Holography

Physical Optics



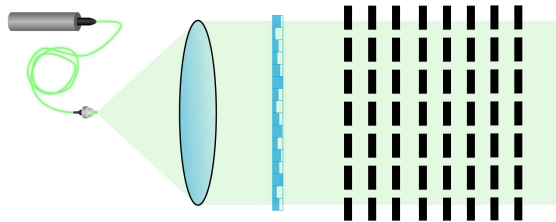
Neural Holography



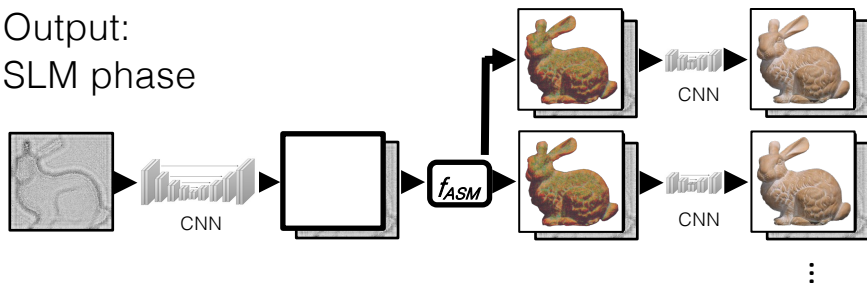
Camera-calibrated Wave Propagation Model

Neural Holography

Physical Optics



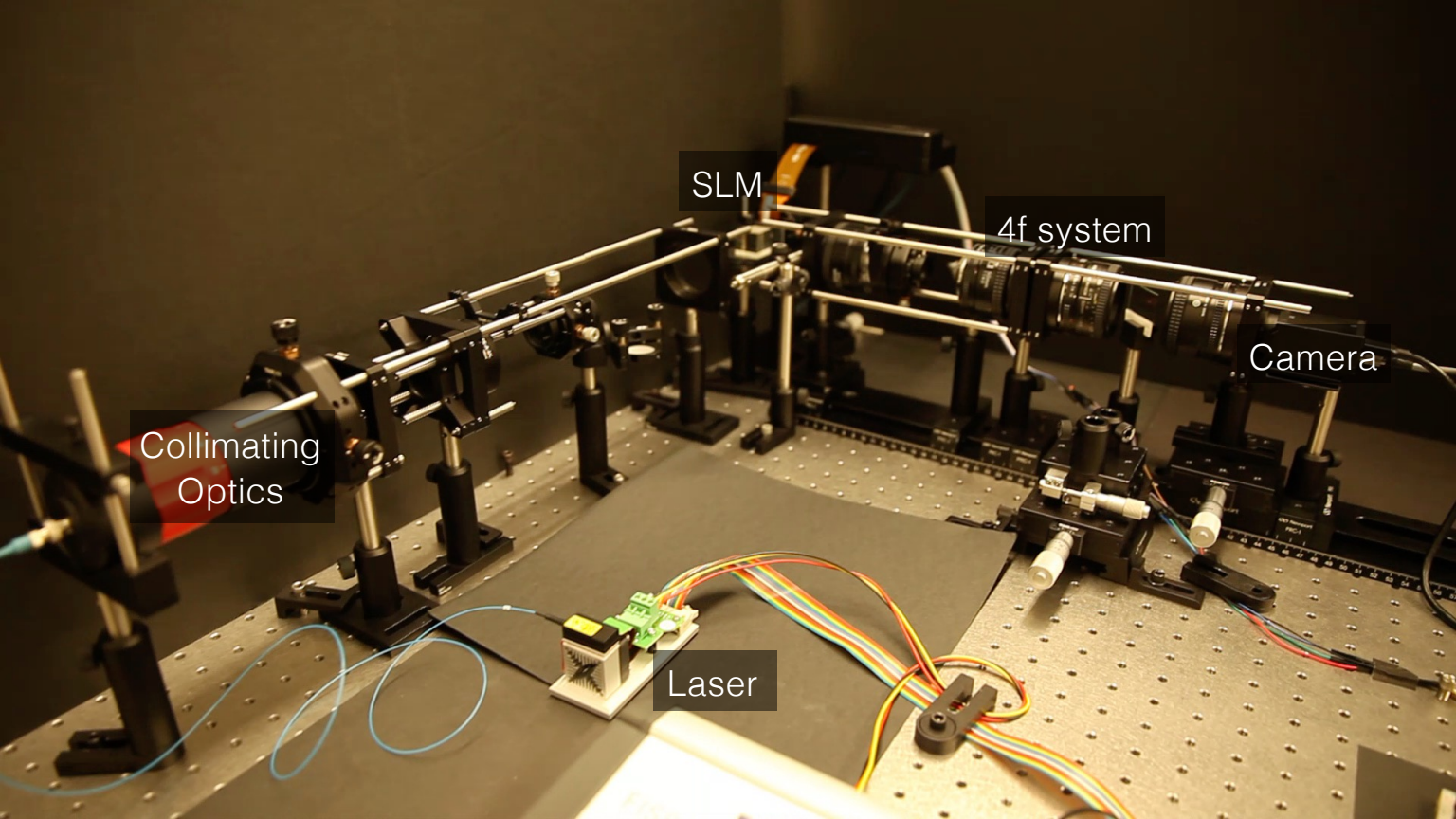
Output:
SLM phase



Camera-calibrated Wave Propagation Model

Input:
2D, 2.5D RGBD, 3D focal
stack, 4D light field, ...





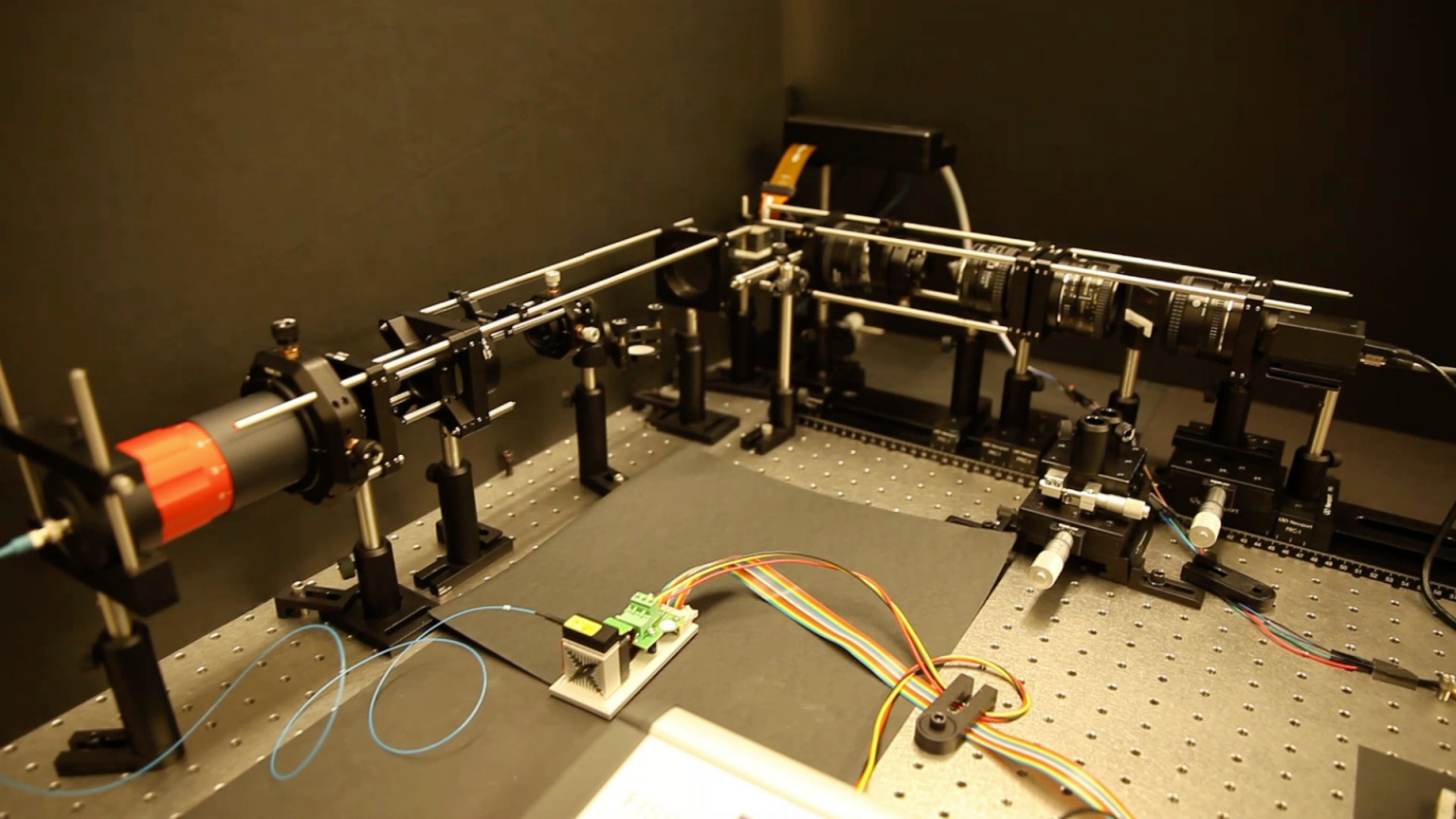
SLM

4f system

Camera

Collimating
Optics

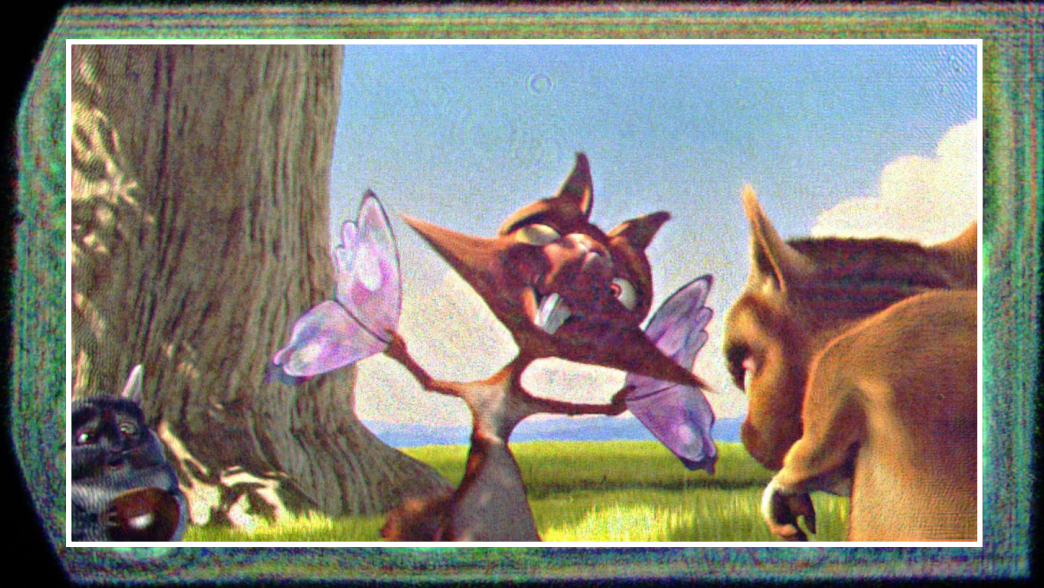
Laser



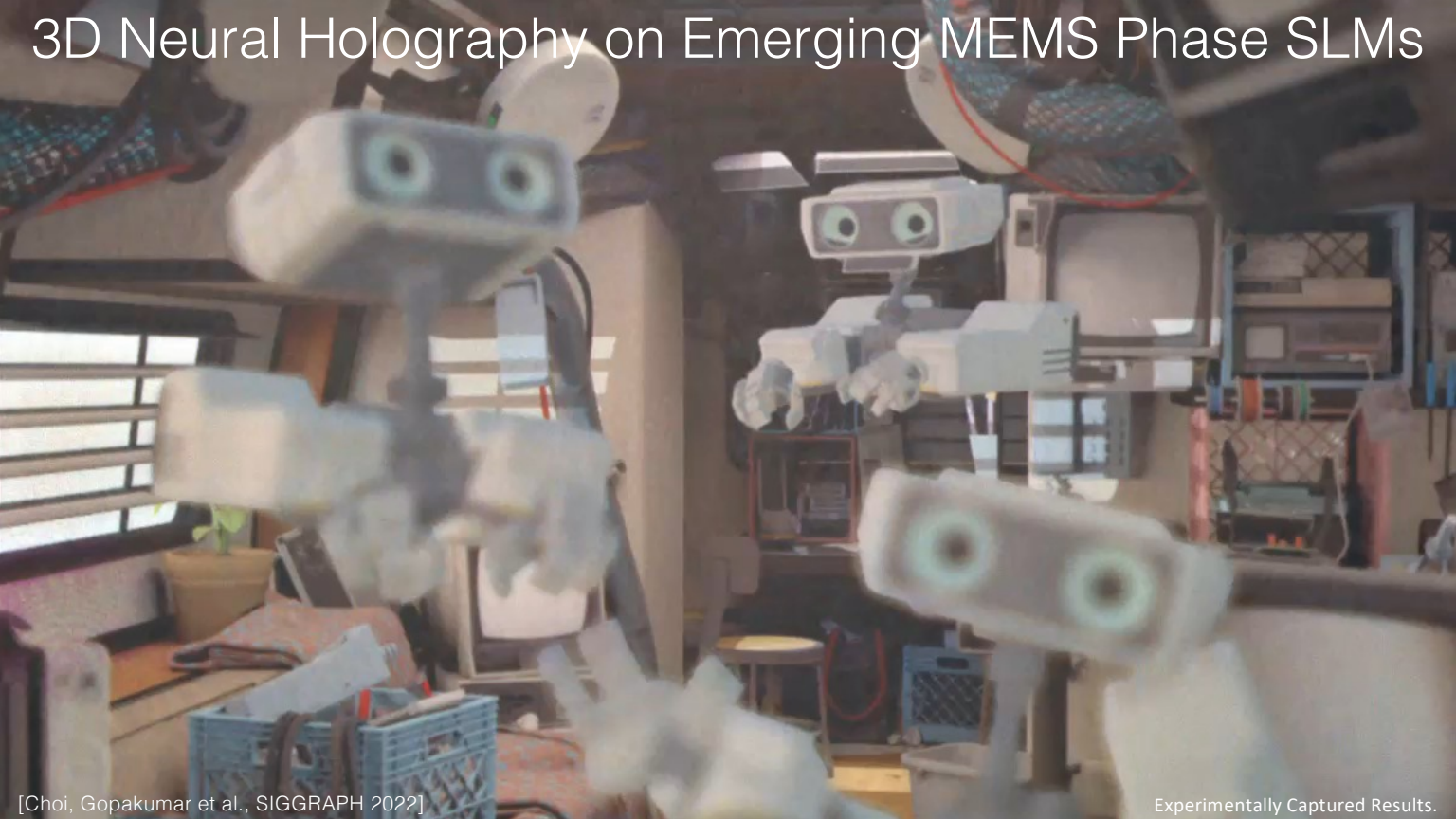
Gerchberg–Saxton



Neural Holography (CITL) 2020 Results



3D Neural Holography on Emerging MEMS Phase SLMs

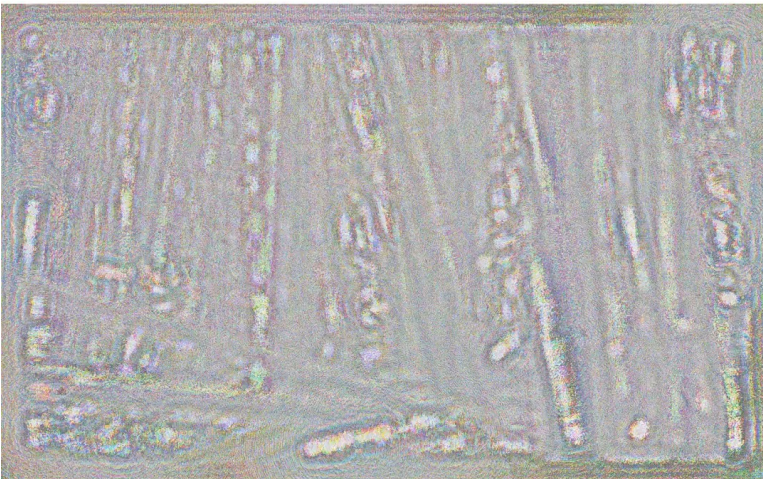


3D Neural Holography on Emerging MEMS Phase SLMs



3D Neural Holography on Emerging MEMS Phase SLMs

Displayed patterns on phase SLM



Holograms captured with our prototype

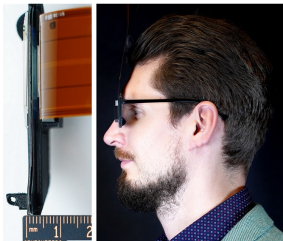


Additional Benefits of Holographic Near-eye Displays

Thin VR Display Form Factors



Maimone et al., SIGGRAPH 2020



Kim et al., SIGGRAPH 2022

Other:

- Light-efficient AR Displays
- Prescription correction (including astigmatism and higher-orders)
- Correcting optical aberrations

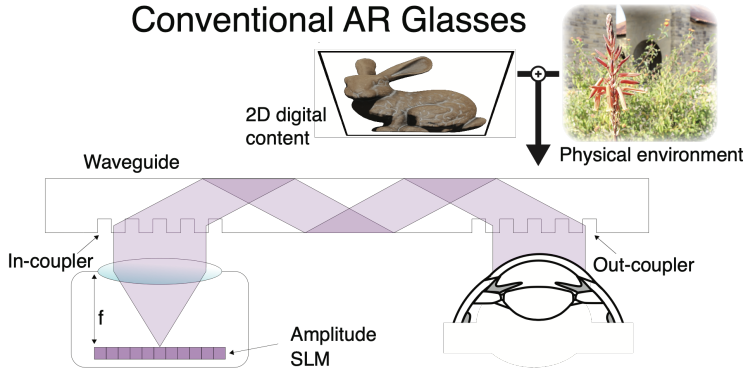
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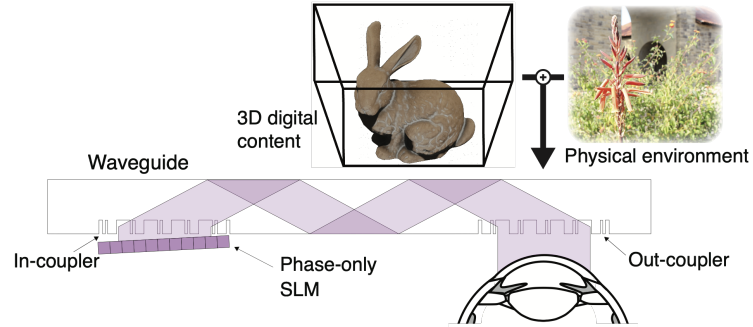
Gopakumar, Lee, ..., Wetzstein, "Full-colour 3D holographic augmented-reality displays with metasurface waveguides", Nature 2024
Photo by Andrew Brodhead

Pairing Holography with Waveguide AR

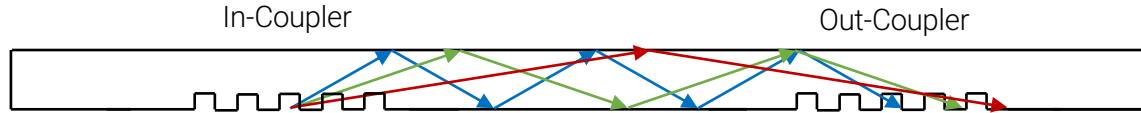
Conventional AR Glasses



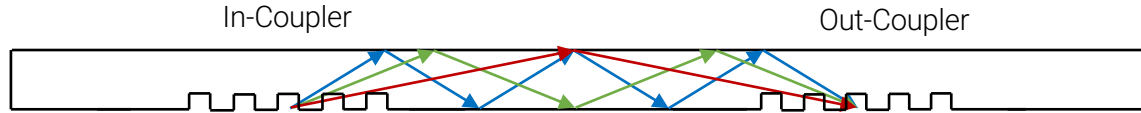
Holographic AR Glasses



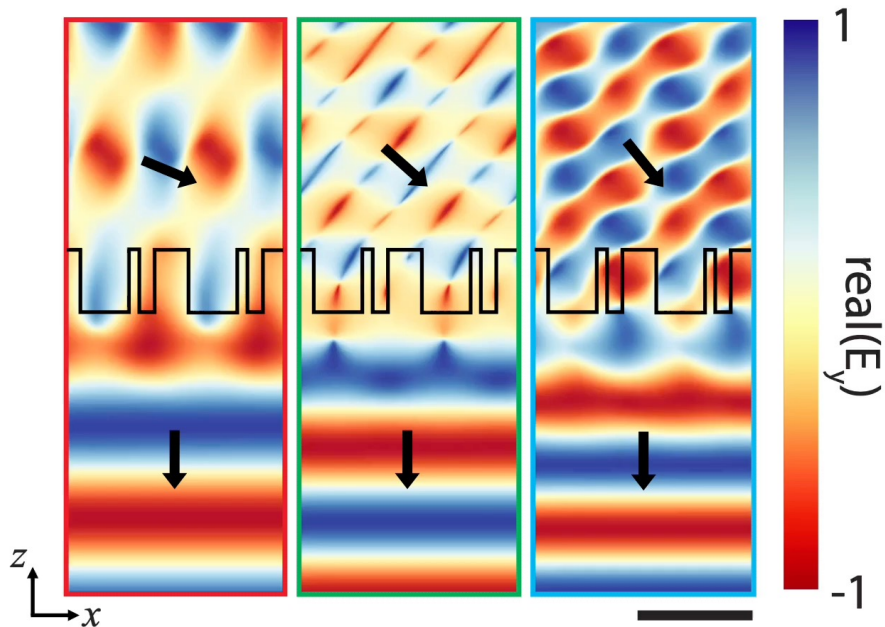
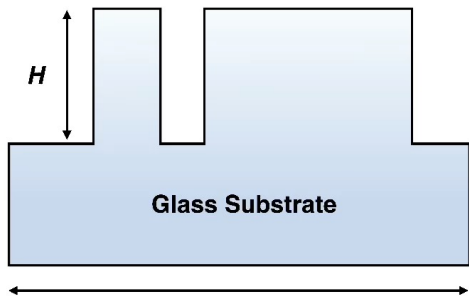
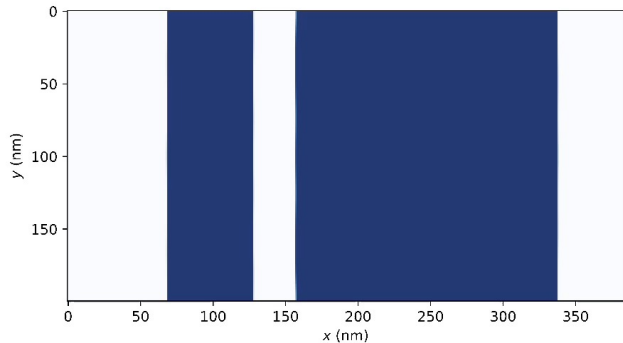
Waveguide Geometry for 3D Holograms



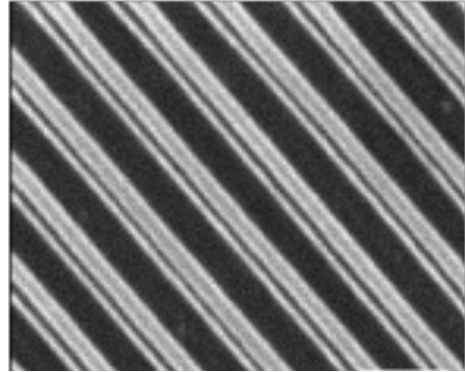
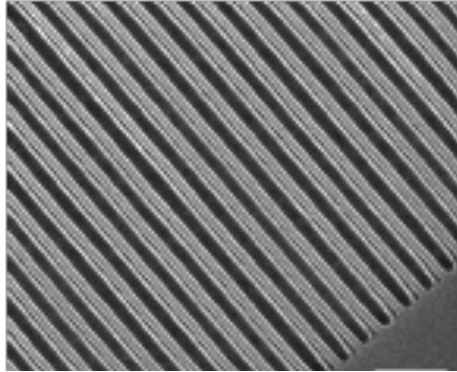
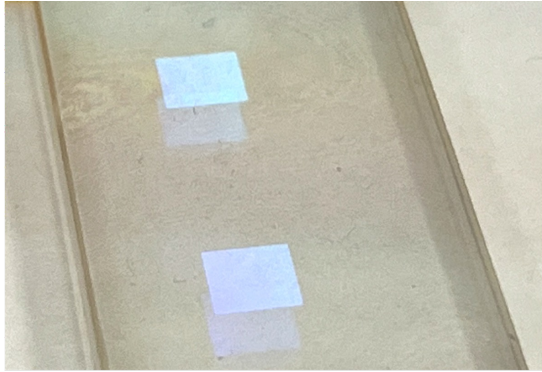
Waveguide Geometry for 3D Holograms



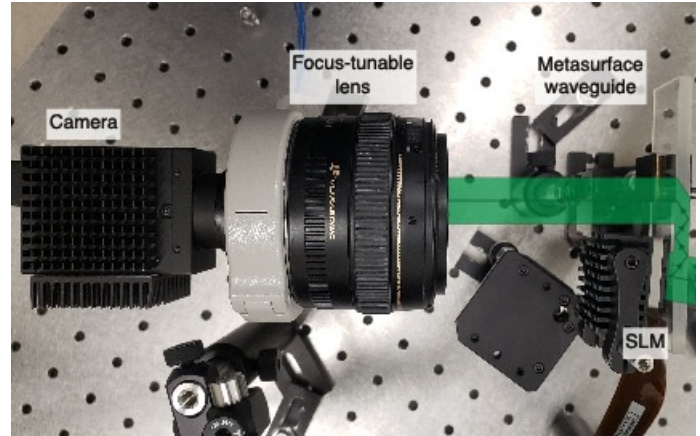
Inverse-designed Metasurface Waveguide



Fabricated Metasurface Waveguide



Metasurface Waveguide Holography Setup



Experimental Results

Conventional Wave Propagation Model

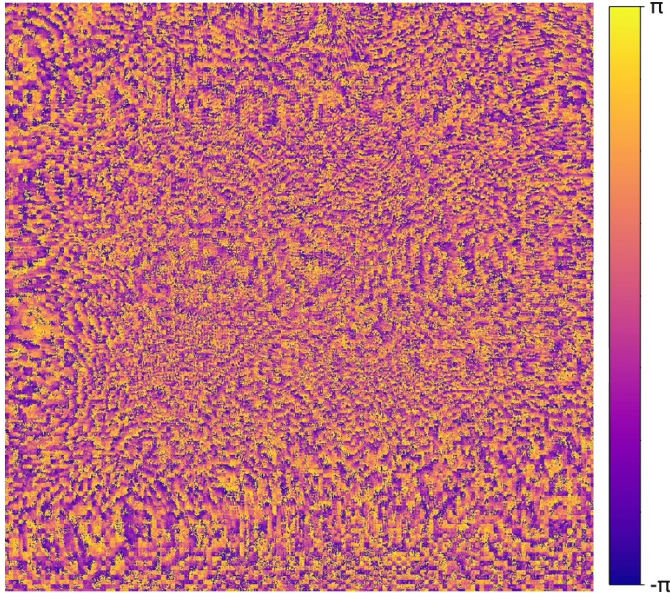


Our Learned Physical Waveguide Model



Experimental 3D Results

SLM Phase Modulation Pattern



View of 3D Hologram

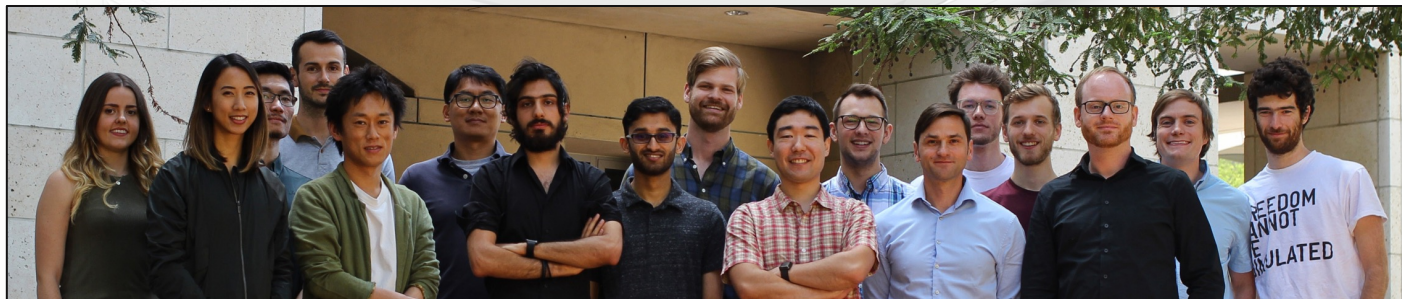


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computationalimaging.org



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References and Further Reading

Phase Retrieval

- R. Gerchberg, W. Saxton, "A Practical Algorithm for the Determination of Phase from Image and Diffraction Plane Pictures", Optik 1972
- J. Fienup, "Phase retrieval algorithms: a comparison", Applied Optics 1982
- ...

Holographic Near-eye Displays

- A. Maimone, A. Georgiou, J.S. Kollin, "Holographic near-eye displays for virtual and augmented reality", ACM SIGGRAPH 2017
- N. Padmanaban, Y. Peng, G. Wetzstein, "Holographic near-eye displays based on overlap-add stereograms", ACM SIGGRAPH Asia 2019
- Y. Peng, S. Choi, N. Padmanaban, G. Wetzstein, "Neural Holography with Camera-in-the-loop Training", ACM SIGGRAPH Asia 2020
- S. Choi, J. Kim, Y. Peng, G. Wetzstein, "Optimizing image quality for holographic near-eye displays with Michelson Holography", OSA Optica 2021
- S. Choi, M. Gopakumar, Y. Peng, J. Kim, G. Wetzstein, "Neural 3D Holography: Learning Accurate Wave Propagation Models for 3D Holographic Virtual and Augmented Reality Displays"
- Y. Peng, S. Choi, J. Kim, G. Wetzstein, "Speckle-free Holography with Partially Coherent Light Sources and Camera-in-the-loop Calibration", Science Advances 2021
- L. Shi, B. Li, C. Kim, P. Kellnhofer, W. Matusik, "Towards real-time photorealistic 3D holography with deep neural networks", Nature 2021

Computational Near-eye Displays with Focus Cues

- See review paper/talk for overview: G. Wetzstein, "Computational Eyeglasses and Near-Eye Displays with Focus Cues", SPIE AR/VR/MR Conference 2020