



Opinionated
Lessons
in Statistics

by Bill Press

#3 Monty Hall

Example: The Monty Hall or Let's Make a Deal Problem



- Three doors
- Car (prize) behind one door
- You pick a door, but don't open it yet
- Monty then opens one of the other doors, always revealing no car (he knows where it is)
- You now get to switch doors if you want
- Should you?
- Most people reason: Two remaining doors were equiprobable before, and nothing has changed. So doesn't matter whether you switch or not.
- Marilyn vos Savant ("highest IQ person in the world") famously thought otherwise (Parade magazine, 1990)
- Monty Hall, the game host, actually understood this (according to a 1991 interview)
 - he is alive in his 90s
 - his daughter is Joanna Gleason, who starred in Sondheim's "Into the Woods"

$H_i =$ car behind door i , $i = 1, 2, 3$

Wlog, you pick door 2 (relabeling).

Wlog, Monty opens door 3 (relabeling).

$$P(H_i|O3) \propto P(O3|H_i)P(H_i)$$

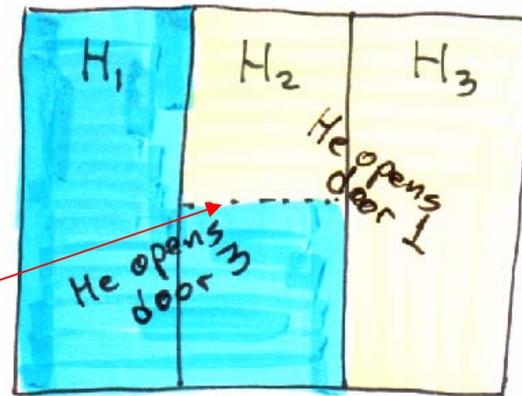
“Without loss of generality...”

$$P(H_1|O3) \propto 1 \cdot \frac{1}{3} = \frac{2}{6}$$

$$P(H_2|O3) \propto \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$P(H_3|O3) \propto 0 \cdot \frac{1}{3} = 0$$

ignorance of Monty's preference
between 1 and 3, so take 1/2



So you should always switch: doubles your chances!

Monty Hall and the Reverend Bayes



- ★ Very important example! Master it.
- ★ $P(H_i) = \frac{1}{3}$ is the “prior probability” or “prior”
- ★ $P(H_i|O3)$ is the “posterior probability” or “posterior”
- ★ $P(O3|H_i)$ is the “evidence factor” or “evidence”
- ★ Bayes says posterior \propto evidence \times prior