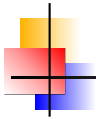


Control-Flow Refinement via Partial Evaluation

Jesús J. Doménech[1]

Samir Genaim[1] and John P. Gallagher[2]

[1]Complutense University of Madrid (Spain)

[2]Roskilde University (Denmark)
[2]IMDEA Software Institute (Spain)

International Workshop on Termination

Oxford. July, 2018

Control-Flow Refinement via Partial Evaluation

1. Control-flow Refinement
2. Partial Evaluation
3. Examples
4. Experiments
5. Conclusions and future work

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

- Two hidden stages.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

- Two hidden stages.
- No Linear Ranking Function.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

- Two hidden stages.
- No Linear Ranking Function.
- Unknown complexity.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

```
0 // int x, y, z;
1 while(x > 0 && y < z)
2   y++;
3
4 while(x > 0 && y >= z)
5   x--;
```

- Two hidden stages.
- No Linear Ranking Function.
- Unknown complexity.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

```
0 // int x, y, z;
1 while(x > 0 && y < z)
2   y++;
3
4 while(x > 0 && y >= z)
5   x--;
```

- Two hidden stages.
- No Linear Ranking Function.
- Unknown complexity.

- Two explicit stages.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

```
0 // int x, y, z;
1 while(x > 0 && y < z)
2   y++;
3
4 while(x > 0 && y >= z)
5   x--;
```

- Two hidden stages.
- No Linear Ranking Function.
- Unknown complexity.

- Two explicit stages.
- Linear Ranking Function.

Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

```
0 // int x, y, z;
1 while(x > 0 && y < z)
2   y++;
3
4 while(x > 0 && y >= z)
5   x--;
```

- Two hidden stages.
- No Linear Ranking Function.
- Unknown complexity.

- Two explicit stages.
- Linear Ranking Function.
- Linear complexity.

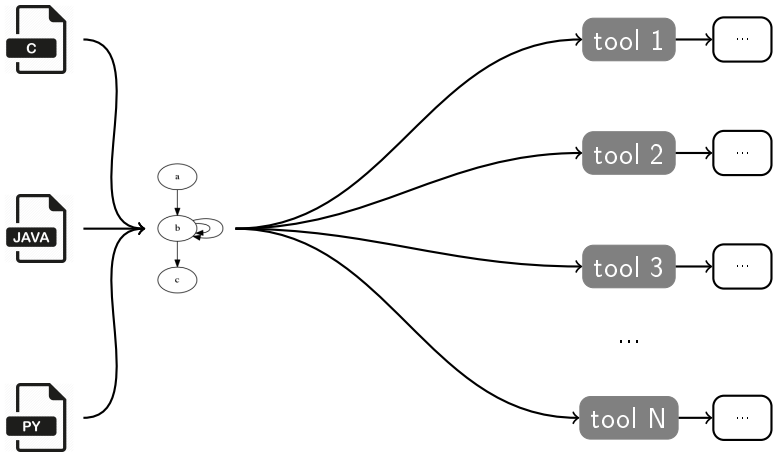
Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

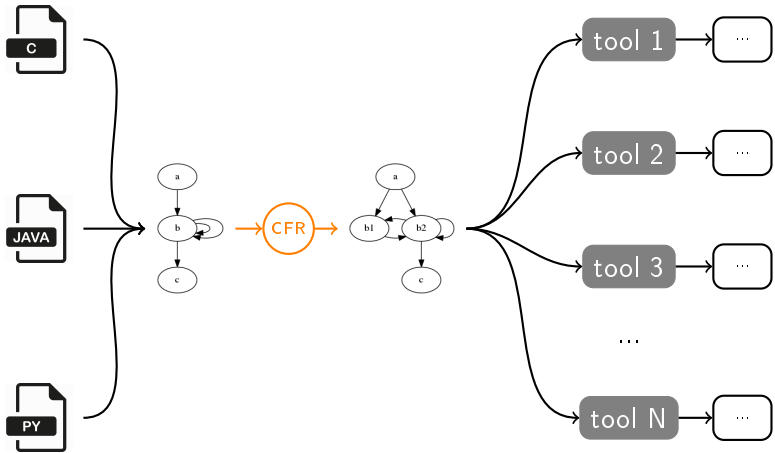
```
0 // int x, y, z;
1 while(x > 0 && y < z)
2   y++;
3
4 while(x > 0 && y >= z)
5   x--;
```

- S. Gulwani et al. Control-flow refinement and progress invariants for bound analysis.
- A. Flores-Montoya. Cost Analysis of Programs Based on the Refinement of Cost Relations. Ph.D. thesis

Control-Flow Refinement



Control-Flow Refinement

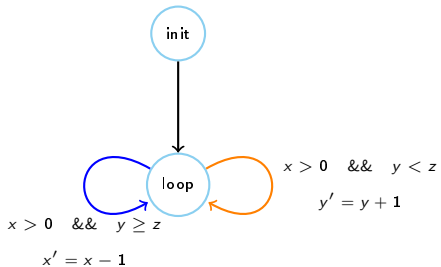


Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

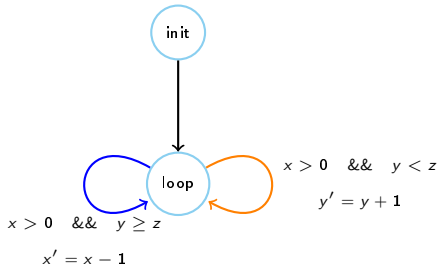
Control-Flow Refinement

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```



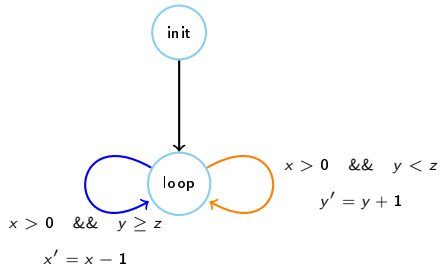
Control-Flow Refinement

$y = 1,\ z = 3$

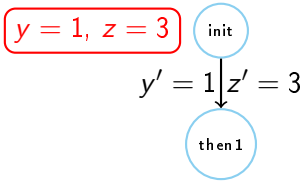


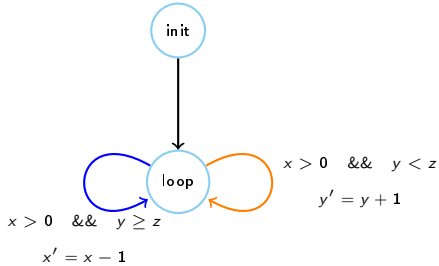
Control-Flow Refinement

$y = 1,\ z = 3$ init

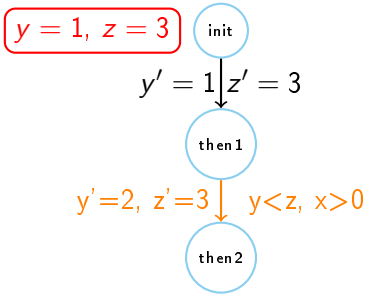


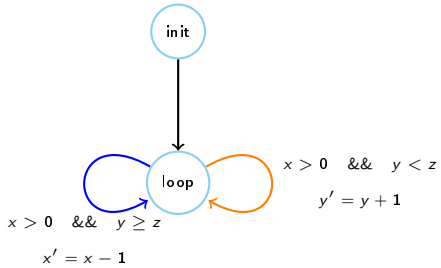
Control-Flow Refinement



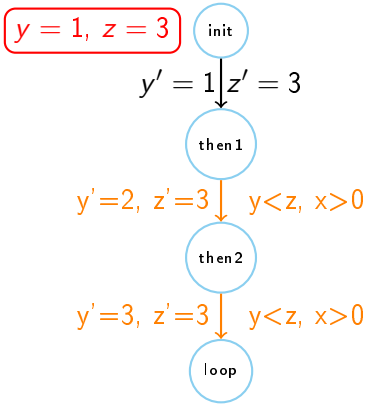


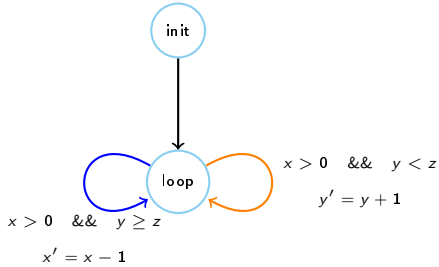
Control-Flow Refinement



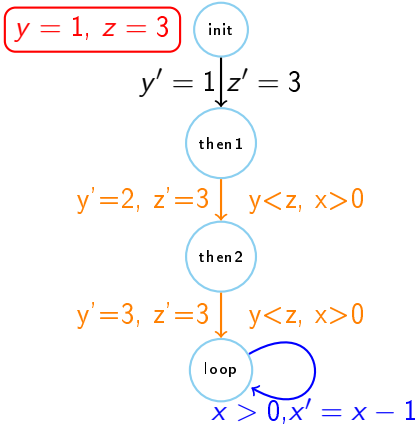


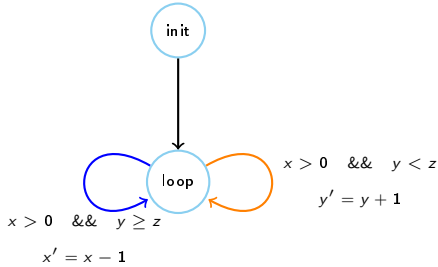
Control-Flow Refinement





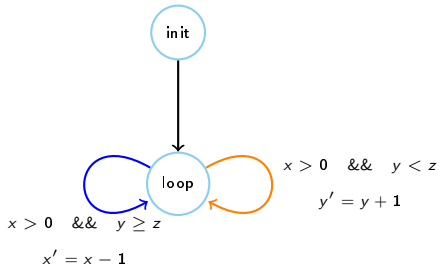
Control-Flow Refinement





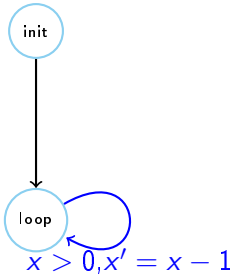
Control-Flow Refinement

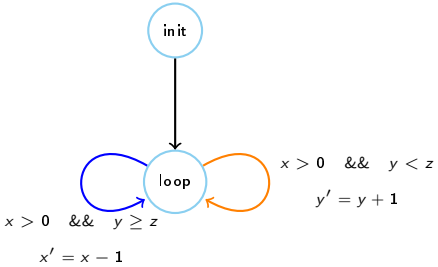
$y = 3,\ z = 1$



Control-Flow Refinement

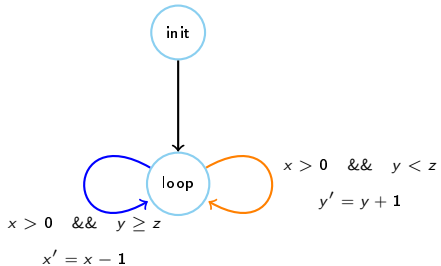
$y = 3,\ z = 1$





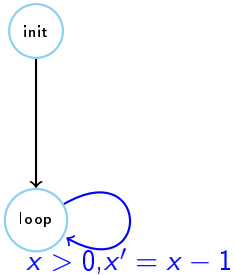
Control-Flow Refinement

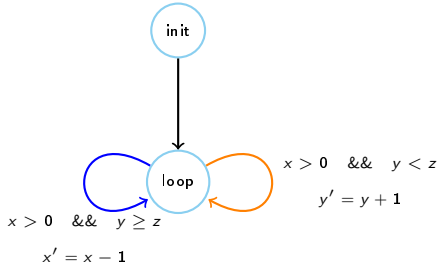
$y \geq z$



Control-Flow Refinement

$y \geq z$





Partial Evaluation

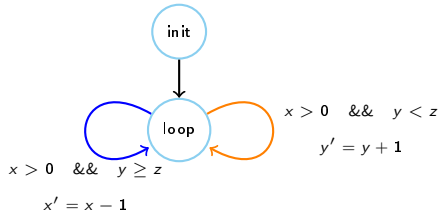
Partial Evaluation (PE) is a program transformation that specializes a program by restricting its meaning in some way.

The PE algorithm used is an instantiation of the described in 1993 by John P. Gallagher. Which has two basic operations:

- `unFold`: extracts one iteration from a loop.
- `abstract`: performs a property-based abstraction with a set of properties. So that, each clause is generalized with the subset of properties implied from the clause.

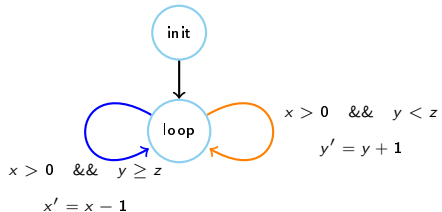
Partial Evaluation: Example

```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```



Partial Evaluation: Example

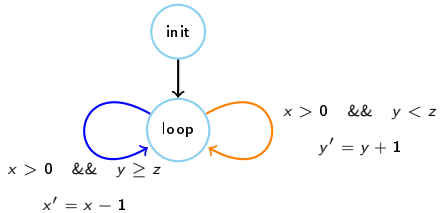
```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```



- Properties: any combination of loop conditions, other conditions, body updates...

Partial Evaluation: Example

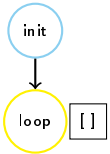
```
0 // int x, y, z;
1 while(x > 0)
2   if(y < z)
3     y++;
4   else
5     x--;
```

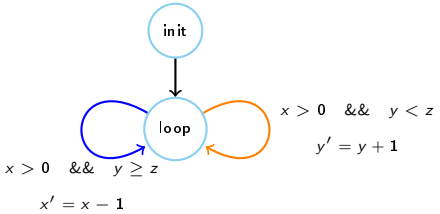


loop:
$\phi_1 = Y \geq Z \quad \&\& \quad X > 0$
$\phi_2 = Y \geq Z$
$\phi_3 = X > 0$
$\phi_4 = Y \geq Z \quad \&\& \quad X > -1$
$\phi_5 = X > -1$
$\phi_6 = Y < Z \quad \&\& \quad X > 0$
$\phi_7 = Y < Z$
$\phi_8 = Y + 1 < Z \quad \&\& \quad X > 0$
$\phi_9 = Y + 1 < Z$

n of loop
s, body

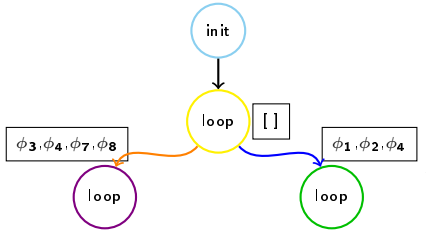
Partial Evaluation: Example

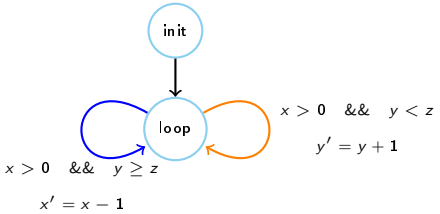




- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

Partial Evaluation: Example





loop:

$\phi_1 = Y \geq Z \quad \&\& \quad X > 0$
$\phi_2 = Y \geq Z$
$\phi_3 = X > 0$
$\phi_4 = Y \geq Z \quad \&\& \quad X > -1$
$\phi_5 = X > -1$
$\phi_6 = Y < Z \quad \&\& \quad X > 0$
$\phi_7 = Y < Z$
$\phi_8 = Y + 1 < Z \quad \&\& \quad X > 0$
$\phi_9 = Y + 1 < Z$

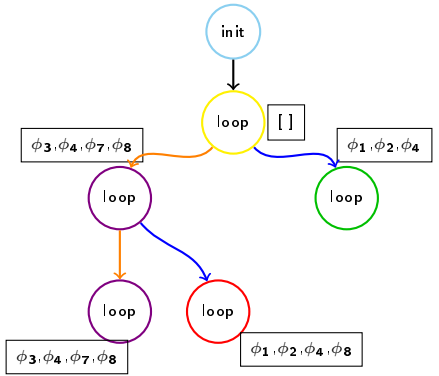
n of loop
s, body

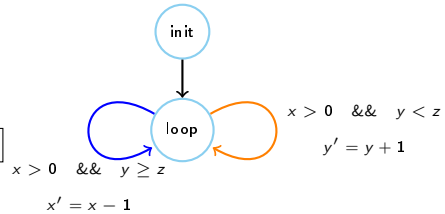
ch leaf node.

set of
g.

(colours) of

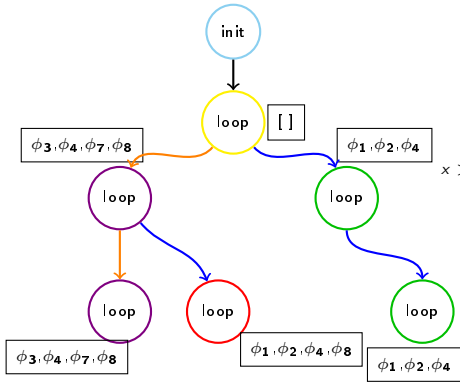
Partial Evaluation: Example

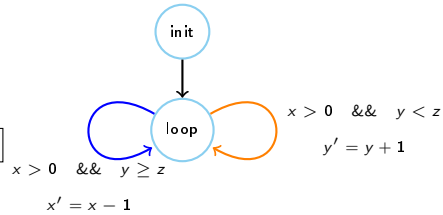




- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

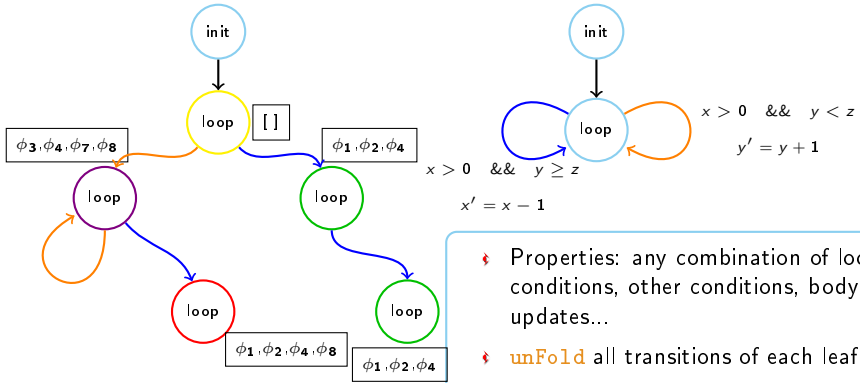
Partial Evaluation: Example





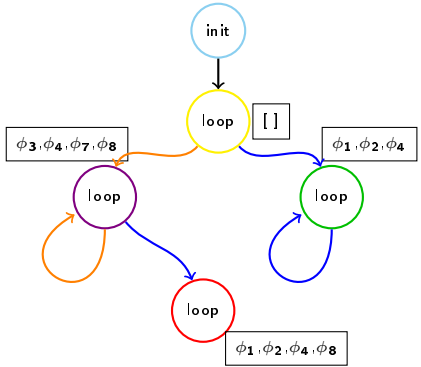
- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

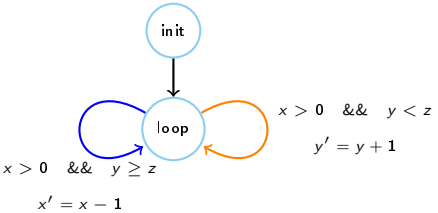
Partial Evaluation: Example



- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

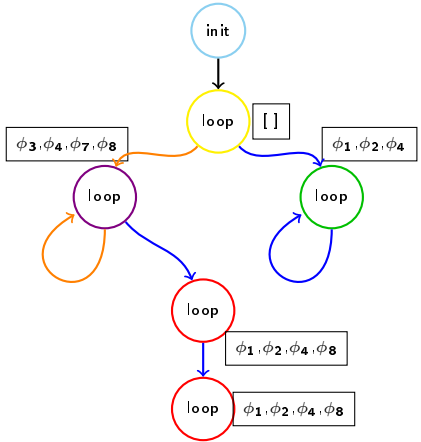
Partial Evaluation: Example

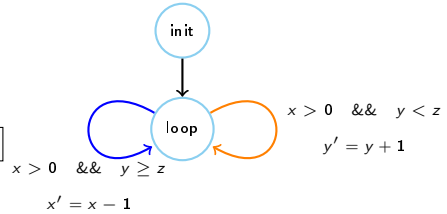




- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

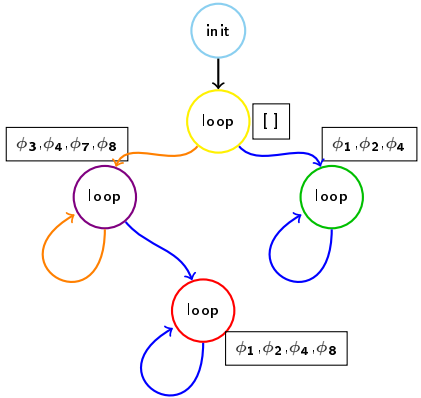
Partial Evaluation: Example

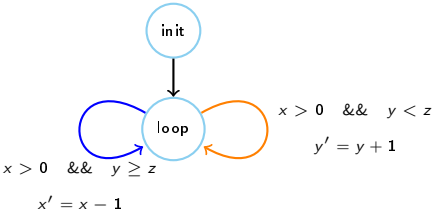




- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

Partial Evaluation: Example

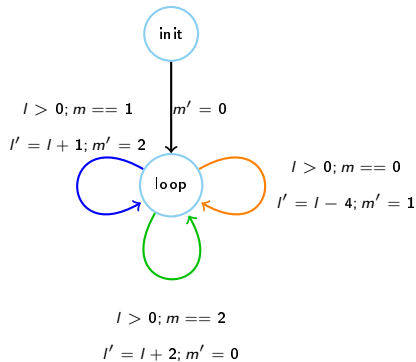




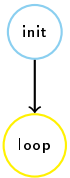
- Properties: any combination of loop conditions, other conditions, body updates...
- `unFold` all transitions of each leaf node.
- `abstract` nodes with same set of properties (colours) holding.
- $2^{\#properties}$ possible versions (colours) of each node.

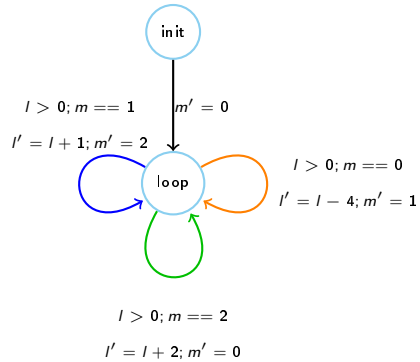
Example: AlternatingGrowReduce

```
AlternatingGrowReduce
0 // uint m, l;
1 m=0;
2 while(l > 0){
3   if(m == 0){
4     l = l - 4;
5     m = 1;
6   }else if(m == 1){
7     l = l + 1;
8     m = 2;
9   }else if(m == 2){
10    l = l + 2;
11    m = 0;
12  }
13}
```

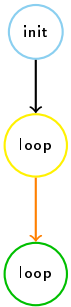


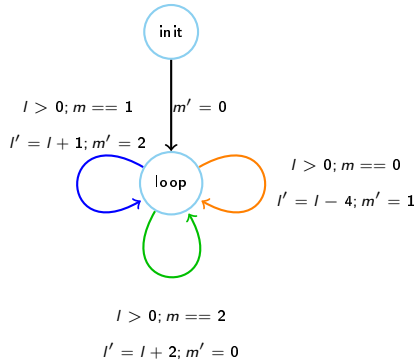
Example: AlternatingGrowReduce



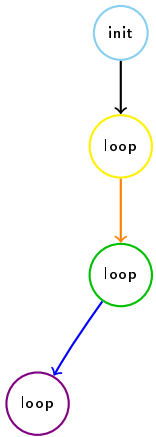


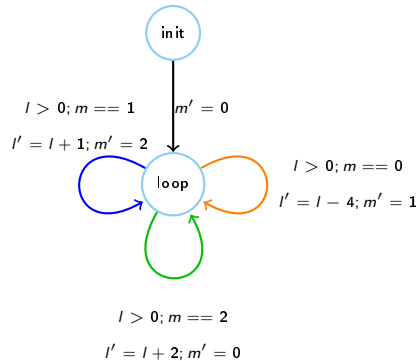
Example: AlternatingGrowReduce



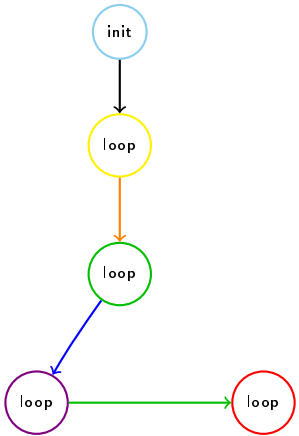


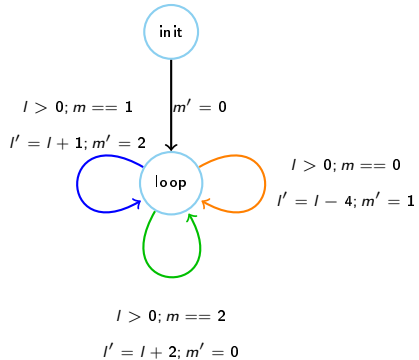
Example: AlternatingGrowReduce



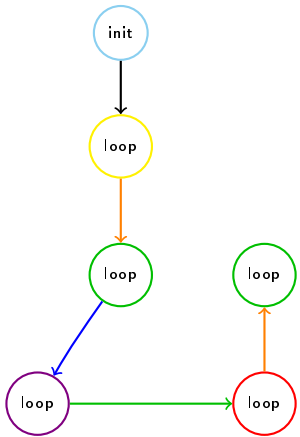


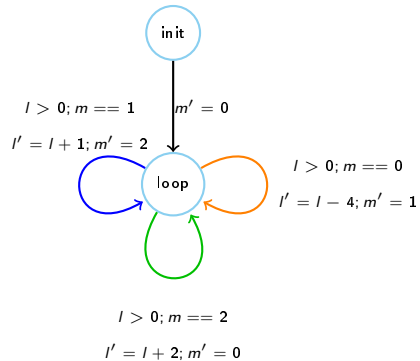
Example: AlternatingGrowReduce



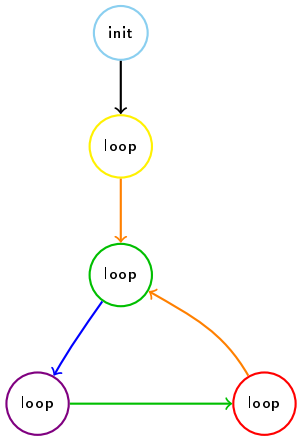


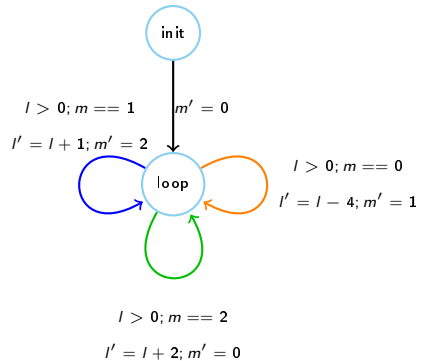
Example: AlternatingGrowReduce





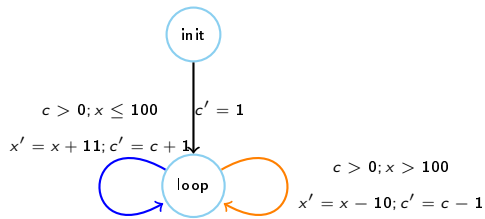
Example: AlternatingGrowReduce



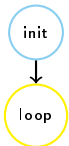


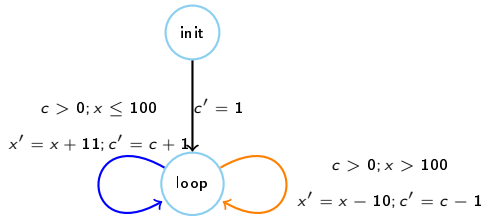
Example: McCarthy91

```
McCarthy 91
0  // uint x, c;
1  c = 1;
2  while(c > 0) {
3    if(x > 100) {
4      x = x - 10;
5      c--;
6    } else {
7      x = x + 11;
8      c++;
9    }
10 }
```

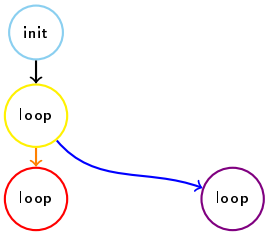


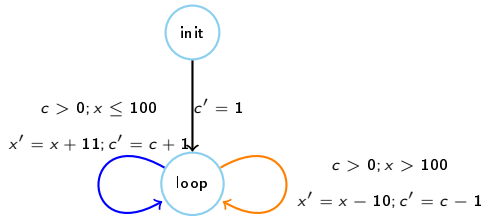
Example: McCarthy91



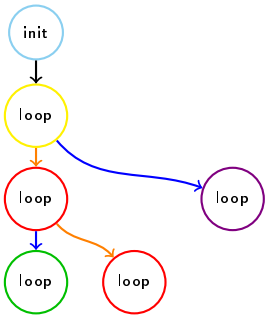


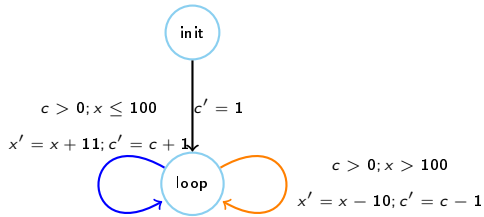
Example: McCarthy91



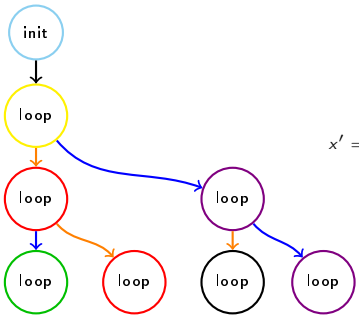


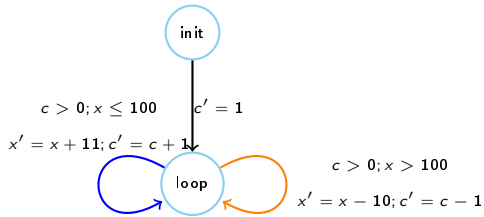
Example: McCarthy91



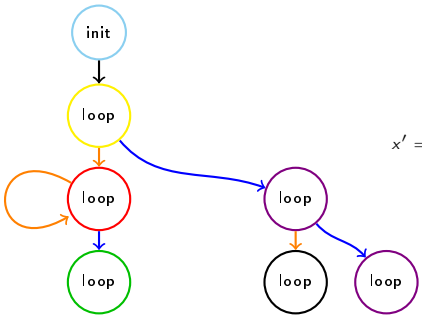


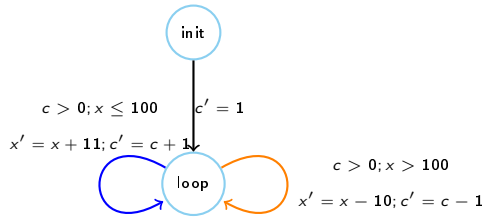
Example: McCarthy91



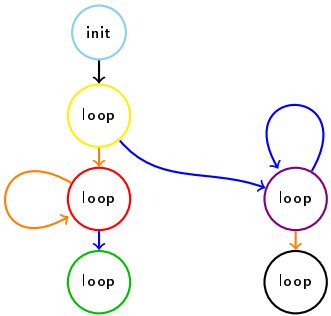


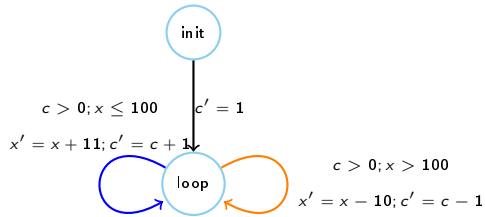
Example: McCarthy91



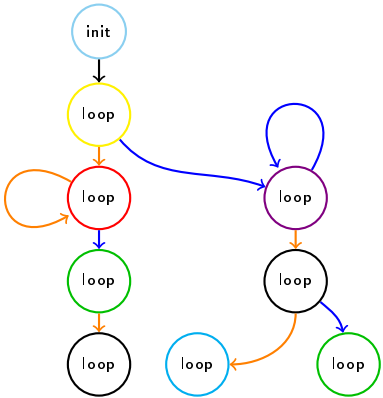


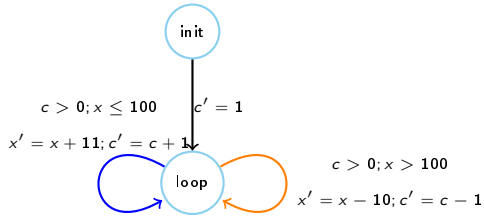
Example: McCarthy91



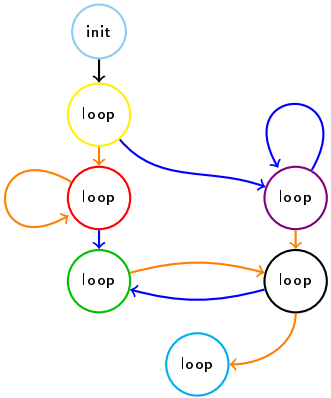


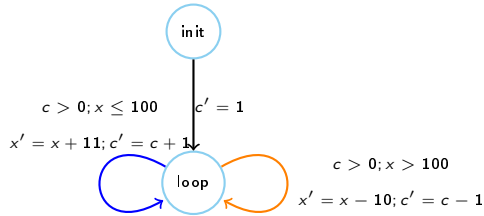
Example: McCarthy91



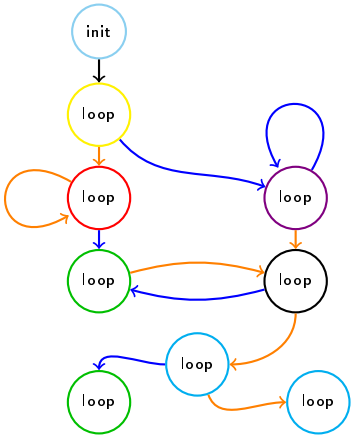


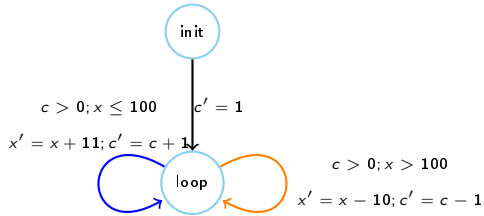
Example: McCarthy91



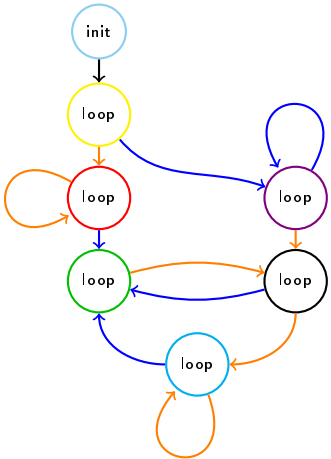


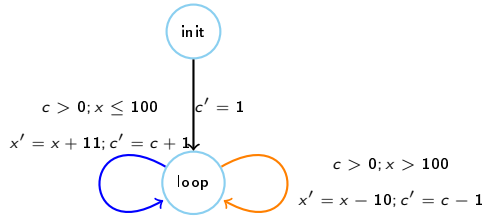
Example: McCarthy91





Example: McCarthy91





Experiments: Tools

We have compared the result of two tools for different purposes with and without refining the program with `Partial Evaluation`.

1. `Koat`: Cost analysis tool for integer transition systems. Marc Brockschmidt et al. *"Analyzing runtime and size complexity of integer programs"*.

Experiments: Tools

We have compared the result of two tools for different purposes with and without refining the program with `Partial Evaluation`.

1. `Koat`: Cost analysis tool for integer transition systems. Marc Brockschmidt et al. *"Analyzing runtime and size complexity of integer programs"*.
2. `iRankFinder`: Our termination analysis tool based on ranking functions as termination arguments.

Experiments: Tools

We have compared the result of two tools for different purposes with and without refining the program with `Partial Evaluation`.

1. `Koat`: Cost analysis tool for integer transition systems. Marc Brockschmidt et al. *"Analyzing runtime and size complexity of integer programs"*.
2. `iRankFinder`: Our termination analysis tool based on ranking functions as termination arguments.

Both tools don't use any control-flow refinement technique.

Experiments: Results

Termination (iRankFinder)

	Unknown to Termination	Lexicografic to Linear	Timeouts with PE	Linear to Lexicografic
TPDB[a]	**12**	**24**	9	5
Cofloco[b]	**8**	**13**	1	2

[a] Termination Problem Database (416 problems)

[b] Cofloco Test Set (188 problems)

Experiments: Results

Termination (iRankFinder)

	Unknown to Termination	Lexicografic to Linear	Timeouts with PE	Linear to Lexicografic
TPDB[a]	**12**	**24**	9	5
Cofloco[b]	**8**	**13**	1	2

Cost (Koat)

	Unknown to Bounded	Better Complexity Class	Timeouts with PE	Worse Complexity Class
TPDB[a]	**4**	**0**	2	3
Cofloco[b]	**10**	**10**	3	3

[a] Termination Problem Database (416 problems)

[b] Cofloco Test Set (188 problems)

Conclusions

- We have explored the use of PE as a control-flow refinement technique in the context of termination and cost analysis.
- Preliminary experiments show that PE improves both analyses.
- `iRankFinder` with PE integrated is available at: `http://irankfinder.loopkiller.com`

Future Work

- To study several cases where the results are worse:
  - identifying the source for this imprecision.
- To specialize the PE tool we use to the context of termination and cost analysis.
- To measure the effect of using PE on performance:
  - exploring the possibility of applying PE only on parts of the CFG.
- To measure the effect of using PE on other termination and cost analysis tools.
- To explore the use of PE for inferring lower-bounds.