

COMP163 Homework Assignment 3: Due Thursday, October 16, 2025 @ 11:59pm

Reading:

1. Review the links on the course webpage from the calendar of recent classes, as well as other the pertinent lecture notes.
2. Begin reading up on Duality in the User Guide and both Duality and Least Median of Squares Regression in the lecture notes.
3. Read section 6.5 (pp. 143-144) in the text and then other parts of Chapter 6 if interested.
4. Begin reading up on Topological Sweep in the lecture notes.

General Information: When describing an algorithm, do not forget to analyse its running time and explain why the algorithm is correct. Although you may discuss these problems in the preliminary stages with others, work submitted should be done individually. If you have any discussions with others (students, friends, TAs, faculty, ...) relative to a homework problem or if you gain information from a written source (including digital sources) other than your own notes from lecture, you are expected to identify your collaborator/source and then be sure that all work submitted is in your own words.

Problems

1. **Diameter and width [O'Rourke]:** Define the *diameter* of a set of points to be the largest distance between any two points in the set.
 - a. Prove that the diameter of a set is achieved by two vertices of the convex hull of the set.
 - b. A *line of support* to a set is a line L that touches the hull and has all points on or to one side of L . Prove that the diameter of a set is the same as the maximum distance between parallel lines of support for the set.
 - c. Two points a and b are called *antipodal* if they admit parallel lines of support: there are parallel lines of support through a and b . Develop an algorithm for enumerating (listing) all antipodal pairs of a set of points in two dimensions. Verify its correctness and analyse its complexity.
 - d. Define the *width* as the minimum distance between parallel lines of support. Develop an algorithm for computing the width of a set of points in two dimensions. Verify its correctness and analyse its complexity.
2. **Minimum-Area Triangle:** Given a set of n points in the plane, the goal is to identify the minimum area triangle defined on those n points.
 - (a) Give a naive $O(n^3)$ algorithm to compute the minimum-area triangle.
 - (b) Given two points p_1 and p_2 and the line they define, describe how to order the remaining $n - 2$ points in increasing order of the area of the triangle each forms with $p_1 p_2$. (Include a figure). How can you characterize the point which forms the smallest area triangle with p_1 and p_2 ?
 - (c) Use duality and line sweep to create an algorithm for the minimum area triangle problem that runs in time $o(n^3)$.
 - (d) **WAIT TO DO THIS PART UNTIL AFTER WE DISCUSS TOPOLOGICAL SWEEP IN CLASS**
Can you then replace line sweep with topological line sweep and improve the running time? If yes, explain how you would do it and the final time complexity. If not, argue why not.
3. **Your Programming Project** Begin thinking about what geometric algorithm/data structure you would like to implement OR what theoretical topic that you might want to investigate. Please write a sentence or two about the type of project that you might want to do
4. **OPTIONAL EXTRA CREDIT: Double Wedges:** Given a set S of n non-vertical line segments specified by the coordinates of their left-endpoints and right endpoints, and given a special line segment t , also specified by its left endpoint and its right endpoint, the goal is to determine whether there is an infinite line that intersects t but does **not** intersection any of the other segments in S , and, if so, give an equation of one such line.

First briefly discuss what you might do to solve this problem in the primal. Then provide the best algorithm that you can to solve this problem in the dual, justify correctness, and analyse complexity.