COMP163 Homework Assignment 4
Due Thursday, October 30, 2025

Reading:
Read the chapter on Voronoi diagrams/Delaunay triangulations in your text. Read the comparable sections in the lecture notes.

NOTE: After Voronoi diagrams, our next topice will be higher dimensional convex hulls.

Problems:

1. The *Gabriel Graph*, GG, of a set S in E^2 is defined as follows: let disk (p_i, p_j) be the circle having $p_i\bar{p}_j$ as a diameter; the Gabriel Graph of S has an edge between p_i and p_j in S if and only if disk (p_i, p_j) contains no point of S in its interior. Show that p_i and p_j have an edge in GG if and only if this edge both appears in the Delaunay triangulation of S and crosses its dual Voronoi edge. Use this to show that the Gabriel graph for any set S can be constructed in $O(n \log n)$ time, and also show the optimality of your algorithm. Give an example of a set S for which the Gabriel Graph and the Delaunay triangulation are NOT the same.

2. Given a set S of n points in the Euclidean plane and a tolerance k:
   (a) describe and analyse an algorithm to report, for each query point q in the plane, the largest circle centered at q that contains no point of S in its interior.
   (b) describe and analyse an algorithm to report, for each pair of query points p, q, a path from p to q contained entirely within the convex hull of S such that no point on the path is closer than k to any point of S, if such a path exists.

3. Submit a rough plan of your programming/visualization or theory project.

   NOTE: by midday on Saturday, we will add comments in Gradescope to what each of you proposed in HW3. Then feel free to followup in office hours next week.

4. OPTIONAL: In the Euclidean space E^d of coordinates $x_1, x_2, ...x_d$, for any real number p such that $1 \leq p \leq \infty$, the L_p-distance of two points q_1 and q_2 is given by the norm:
   $d_p(q_1, q_2) \ = \ (\sum_{j=1}^{d}(|x_j(q_1) - x_j(q_2)|)^p)^{1/p}$.
   So far, we have been studying the L_2-distance in E^2 (the plane).

   Draw a set of 4-5 points at integer grid points and work out each of the parts below on this small sample. Then generalize if possible.

   (a) In the plane, characterize the Voronoi diagram of a set of N points in the L_1-metric.
   (b) Solve the same problem for the L_∞-metric.
   (c) What is the relationship between the Voronoi diagram in the L_1-metric and that in the L_∞-metric?