DRAWING THE UNKNOWN

Maarten Löffler
Utrecht University

Hello!

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I am a computational geometer.

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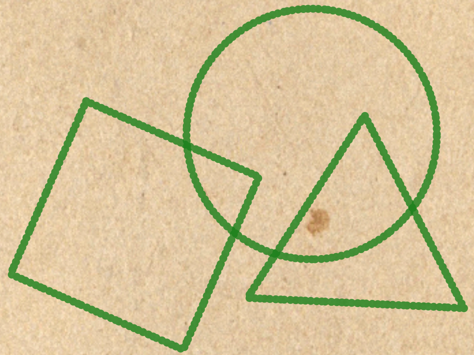

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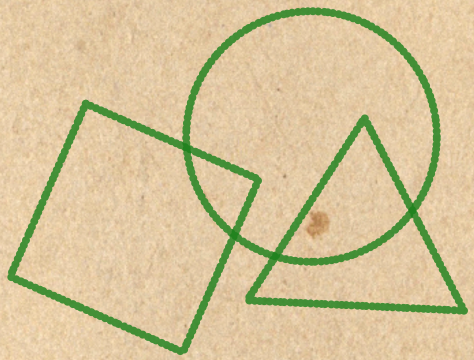

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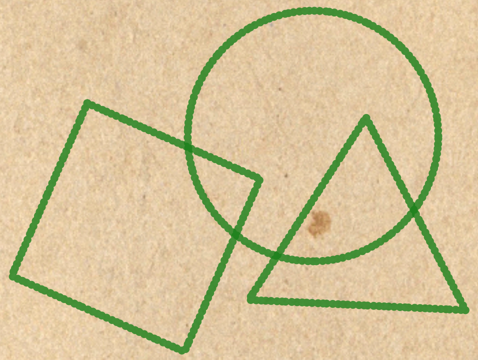

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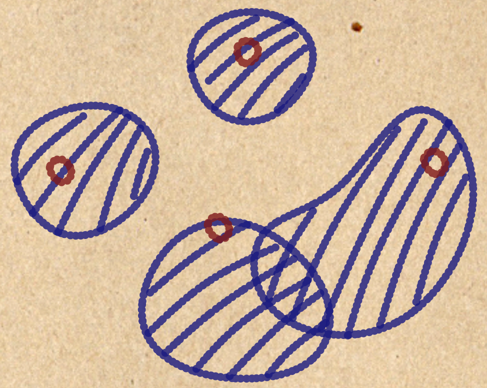

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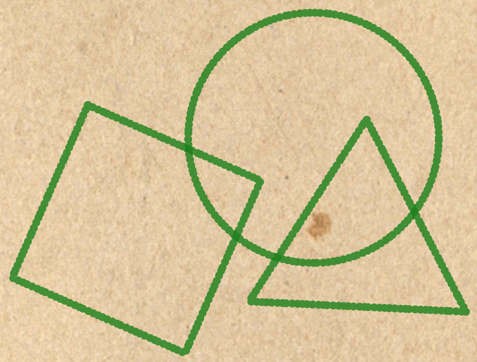

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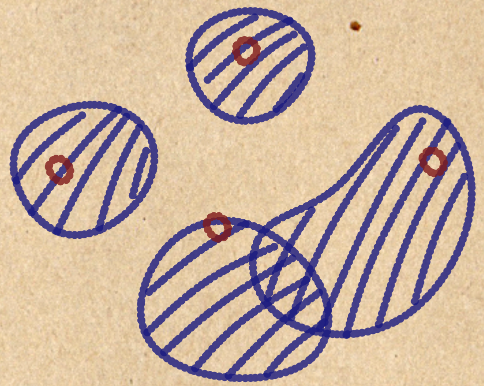

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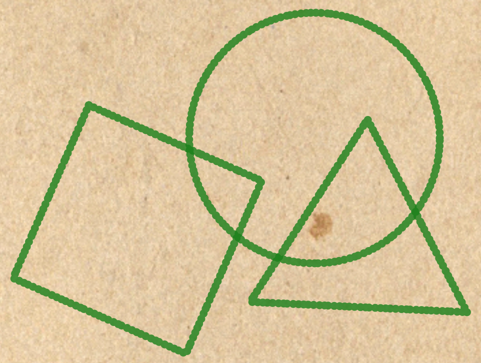

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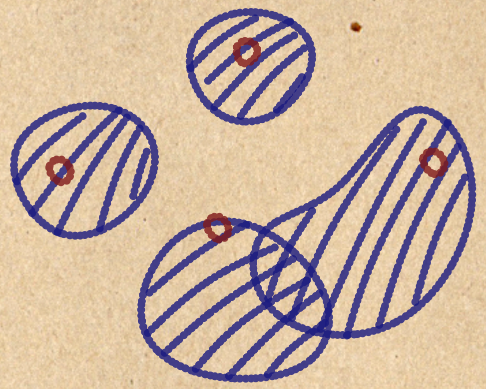

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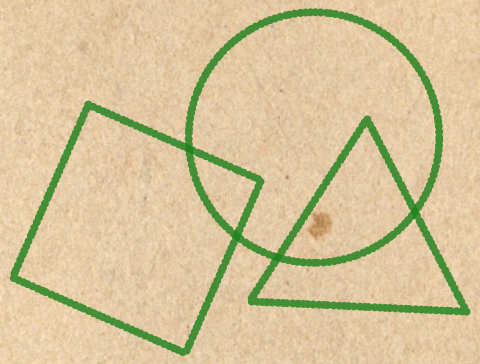

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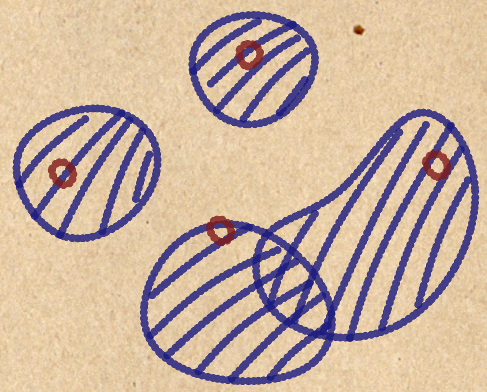

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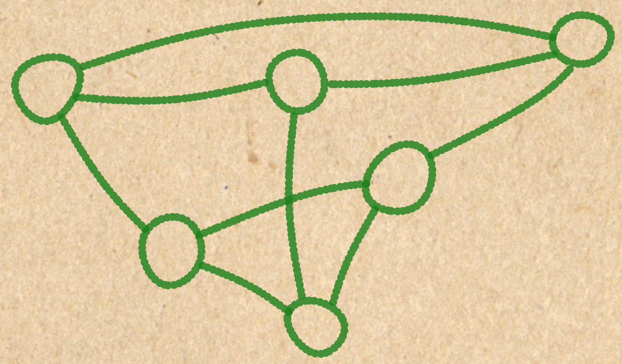

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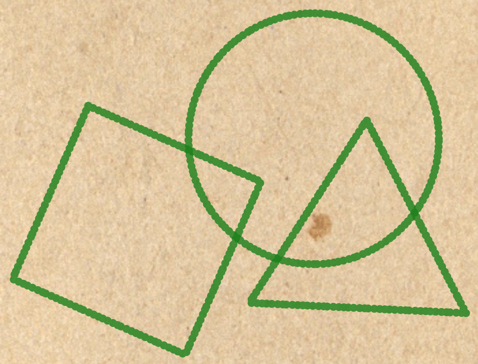

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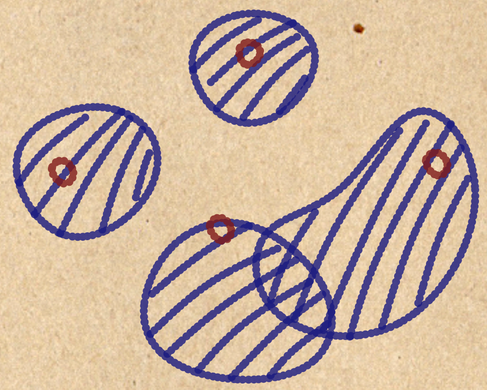

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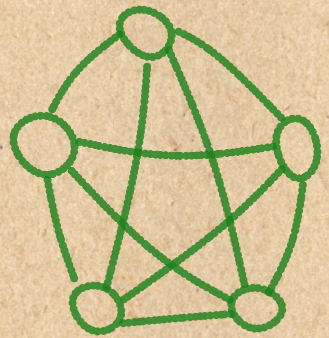

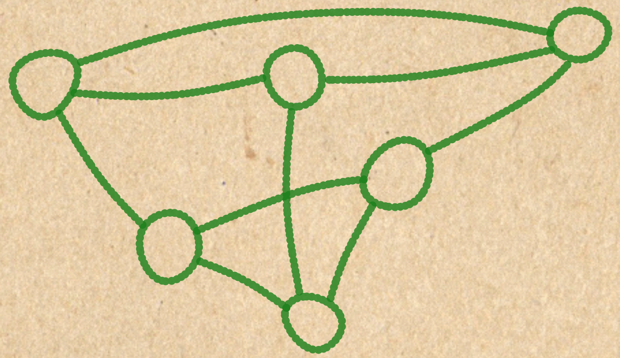

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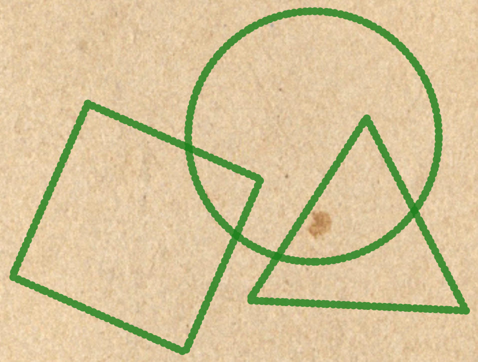

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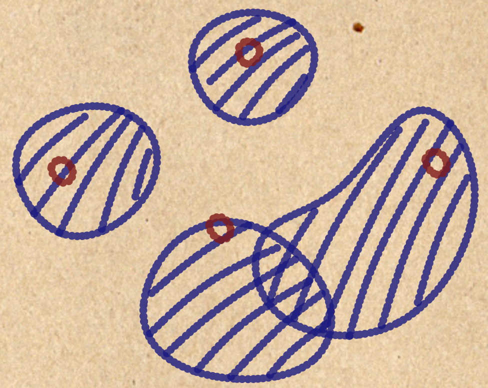

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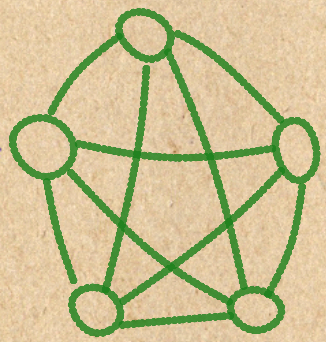

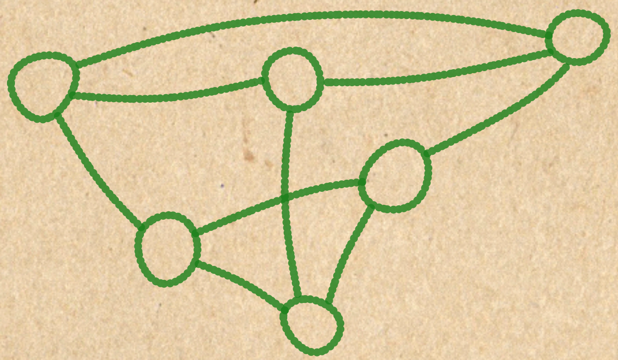

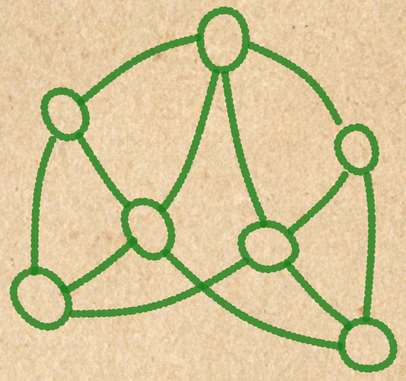

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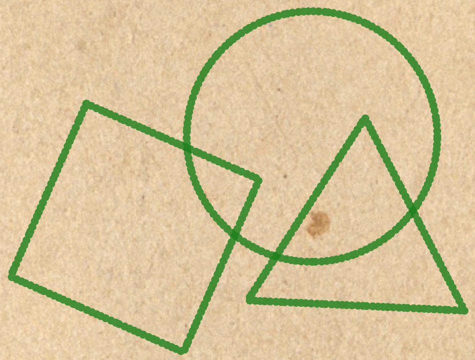

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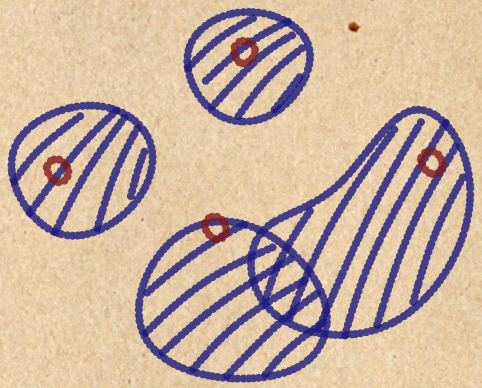

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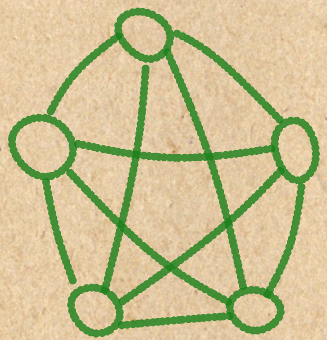

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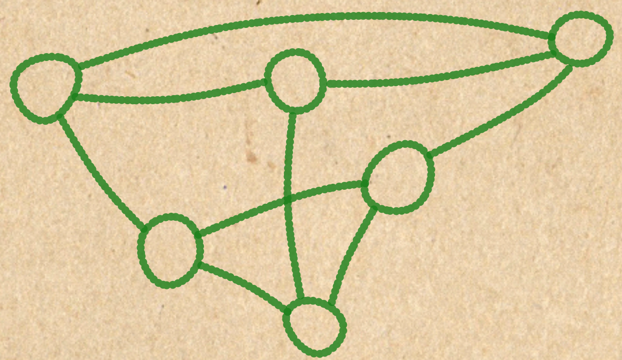

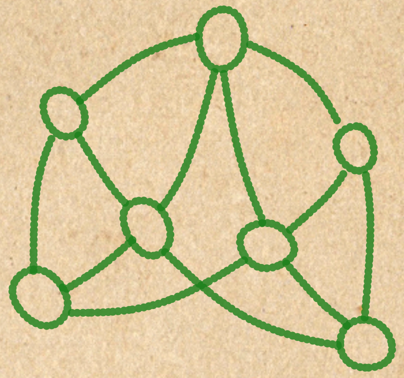

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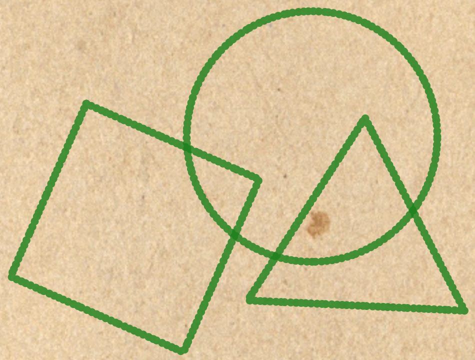

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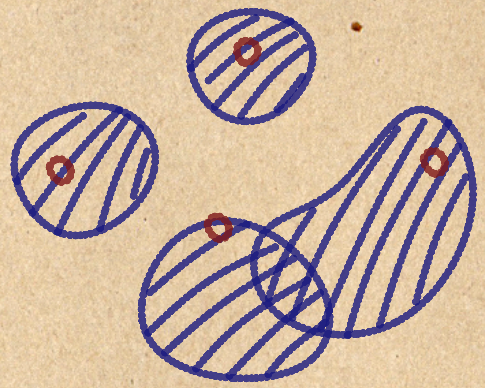

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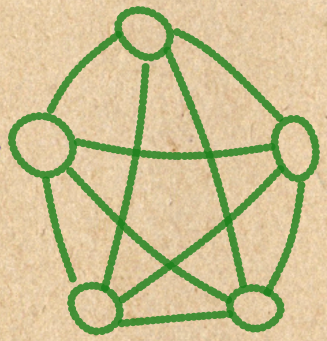

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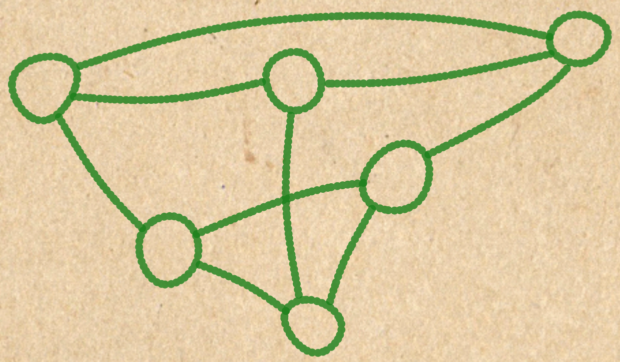

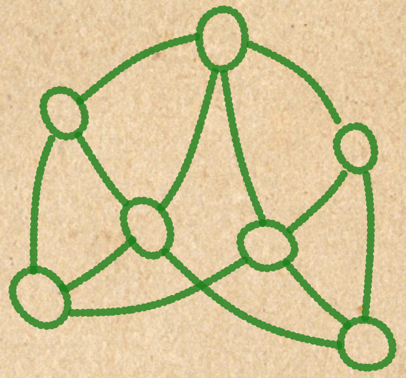

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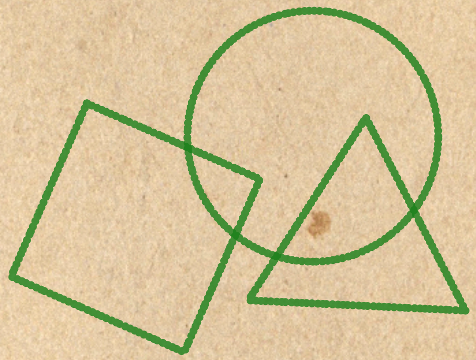

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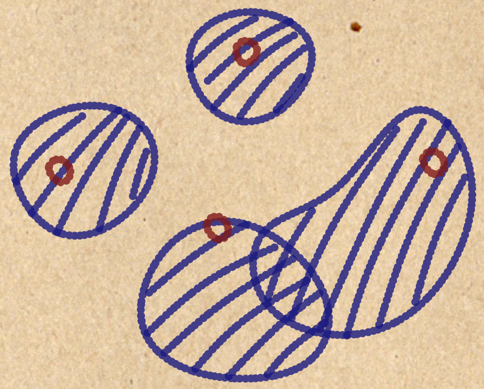

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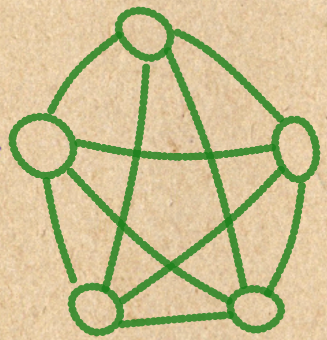

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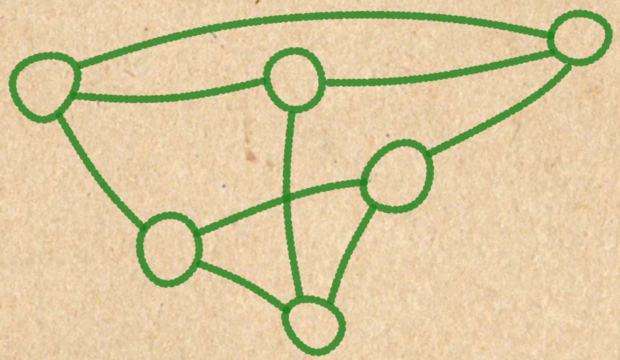

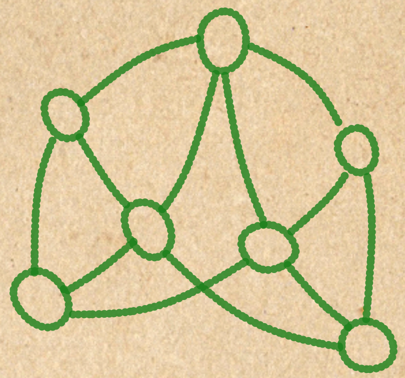

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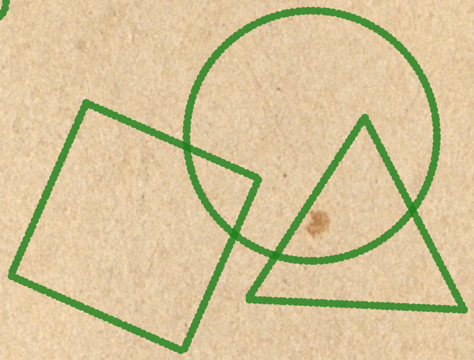

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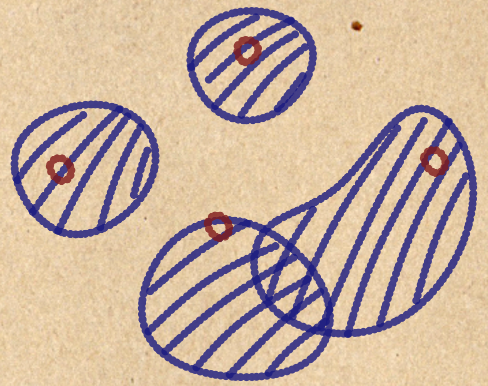

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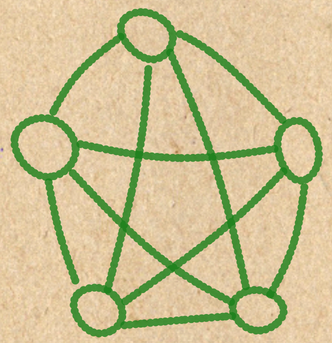

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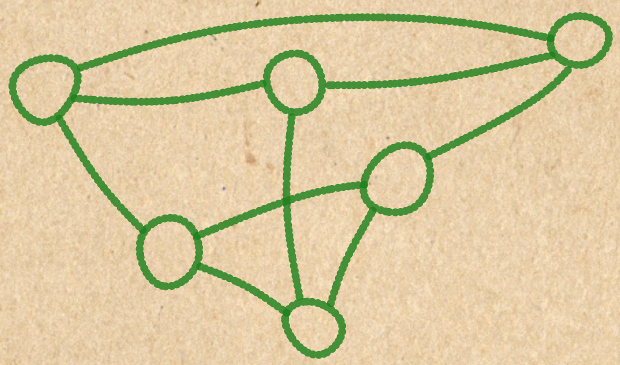

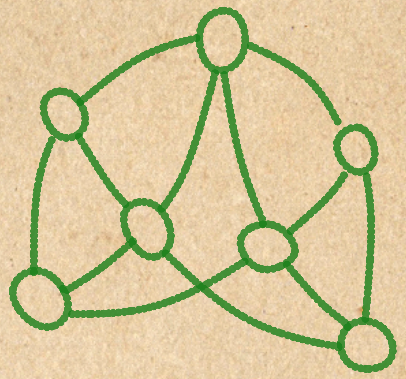

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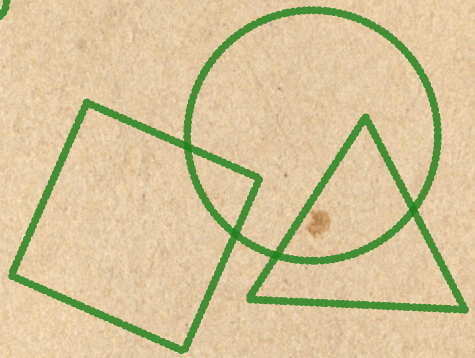

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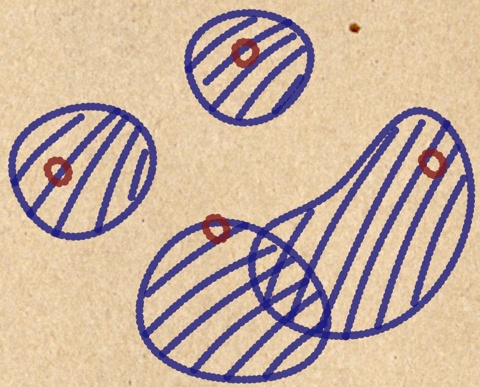

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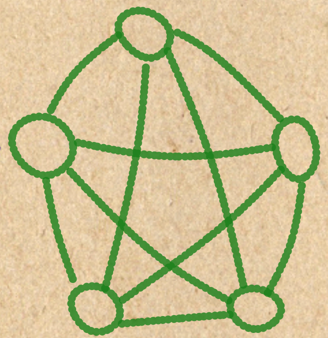

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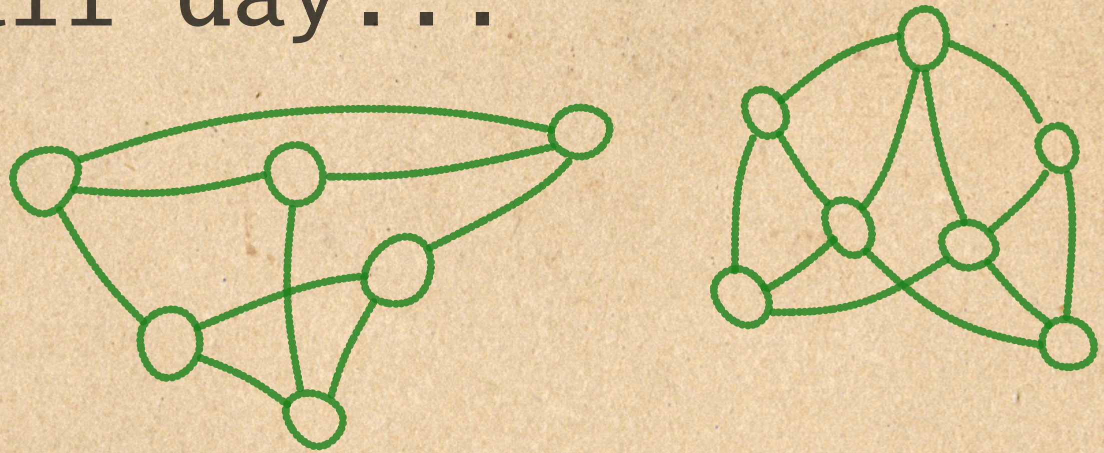

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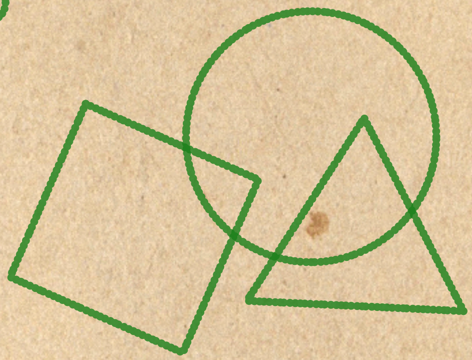

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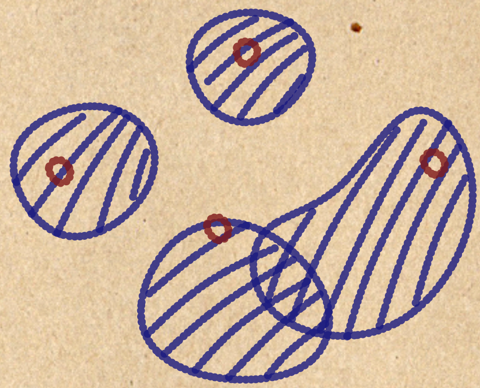

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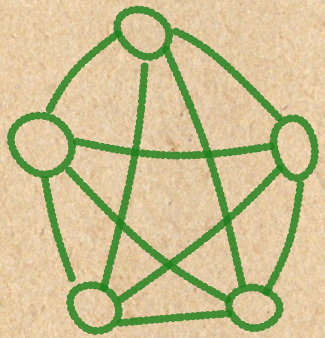

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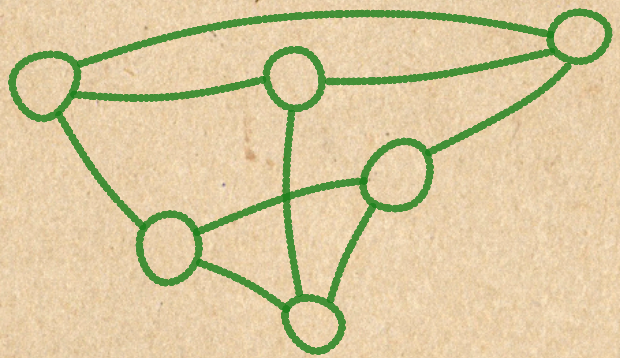

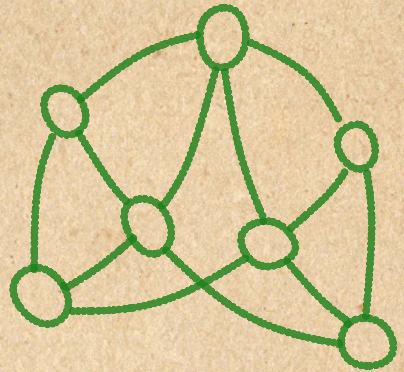

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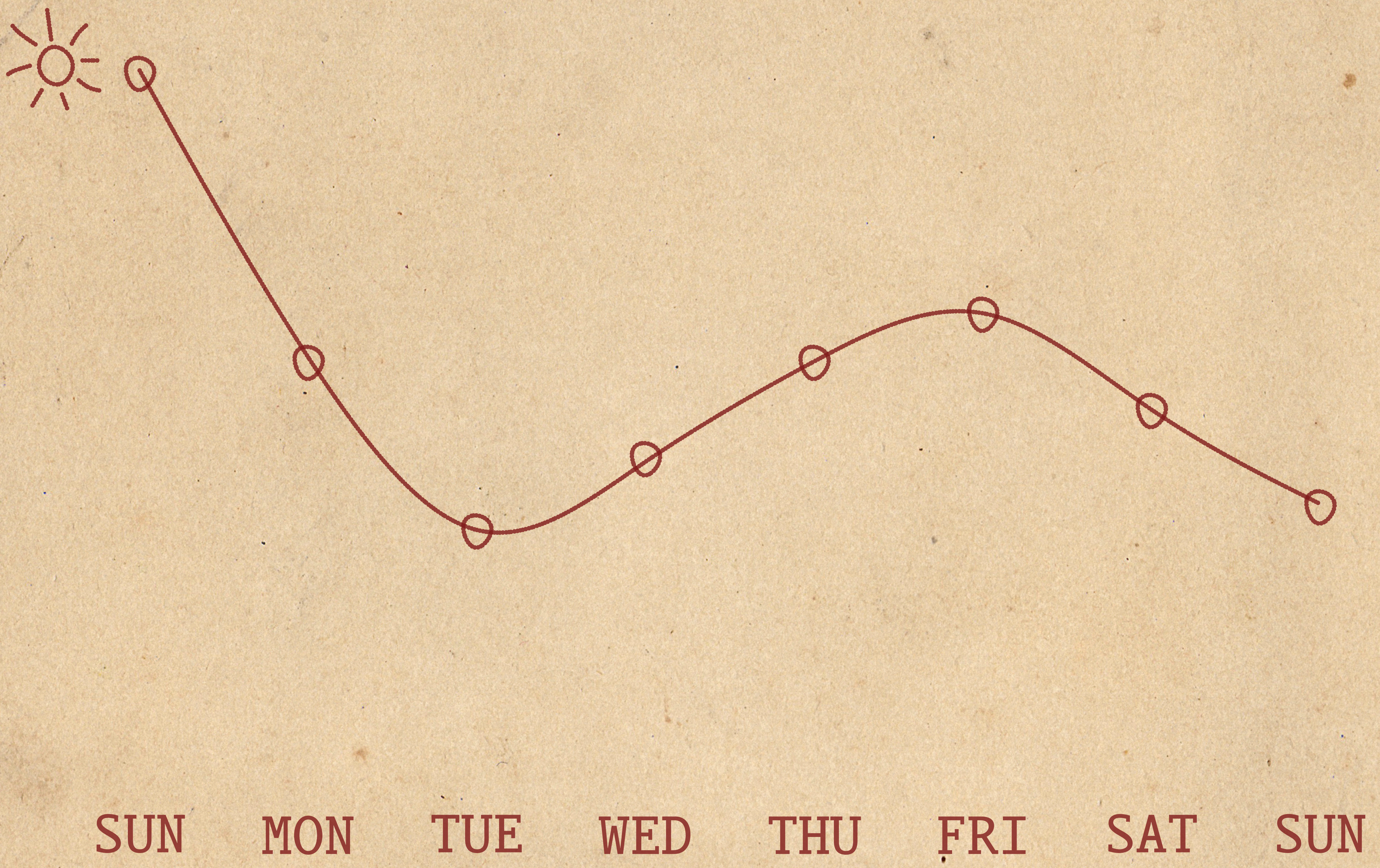

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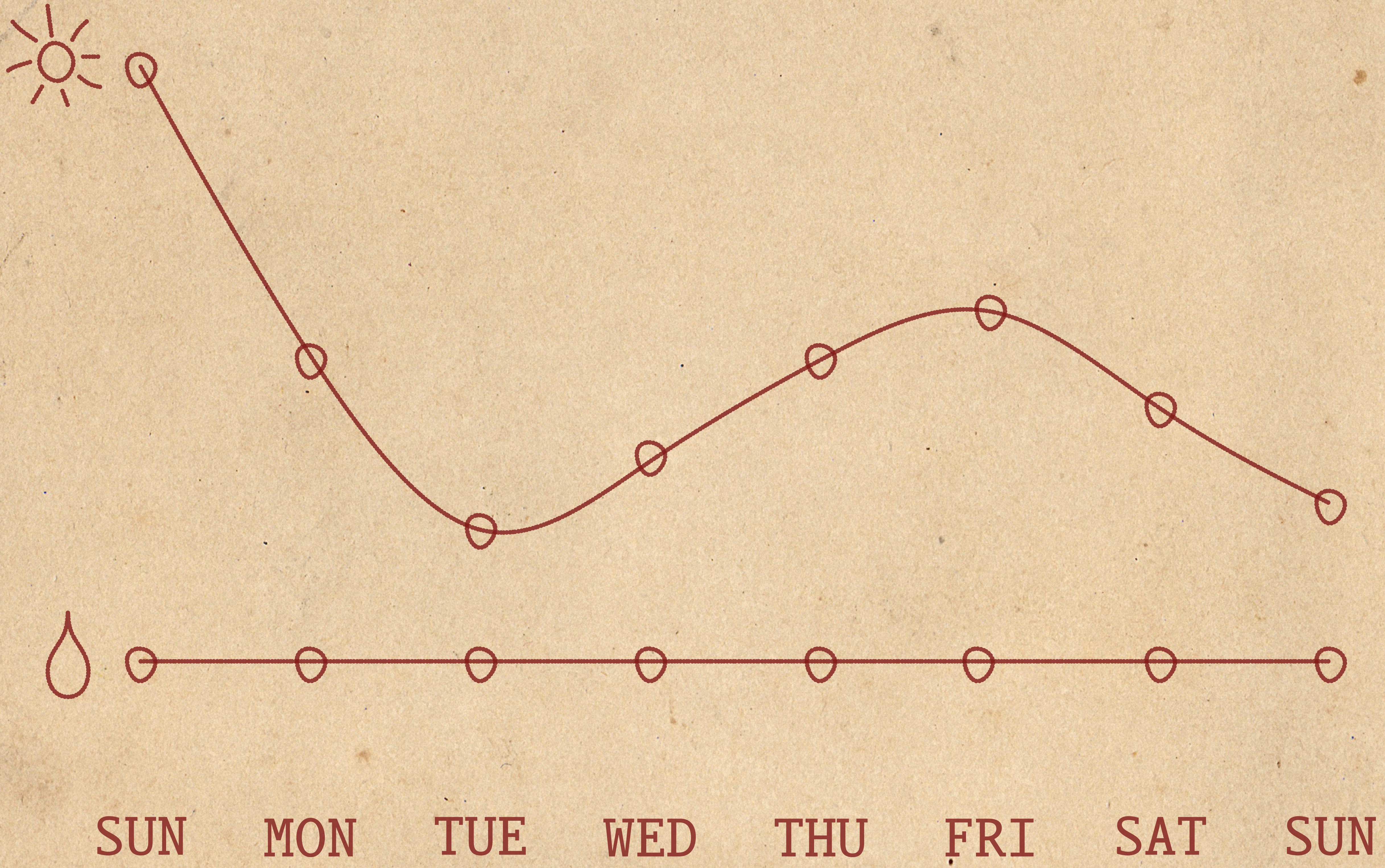

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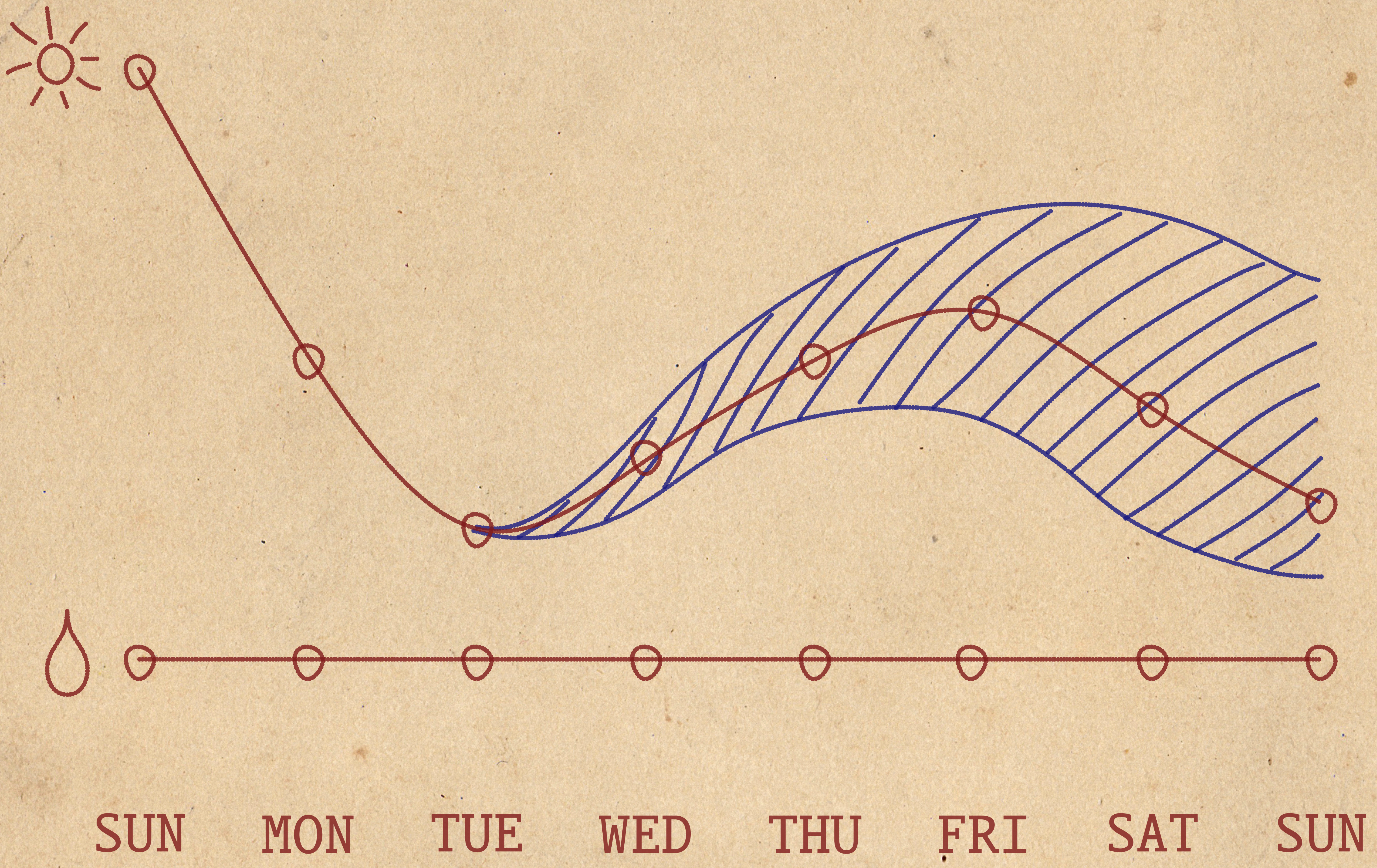

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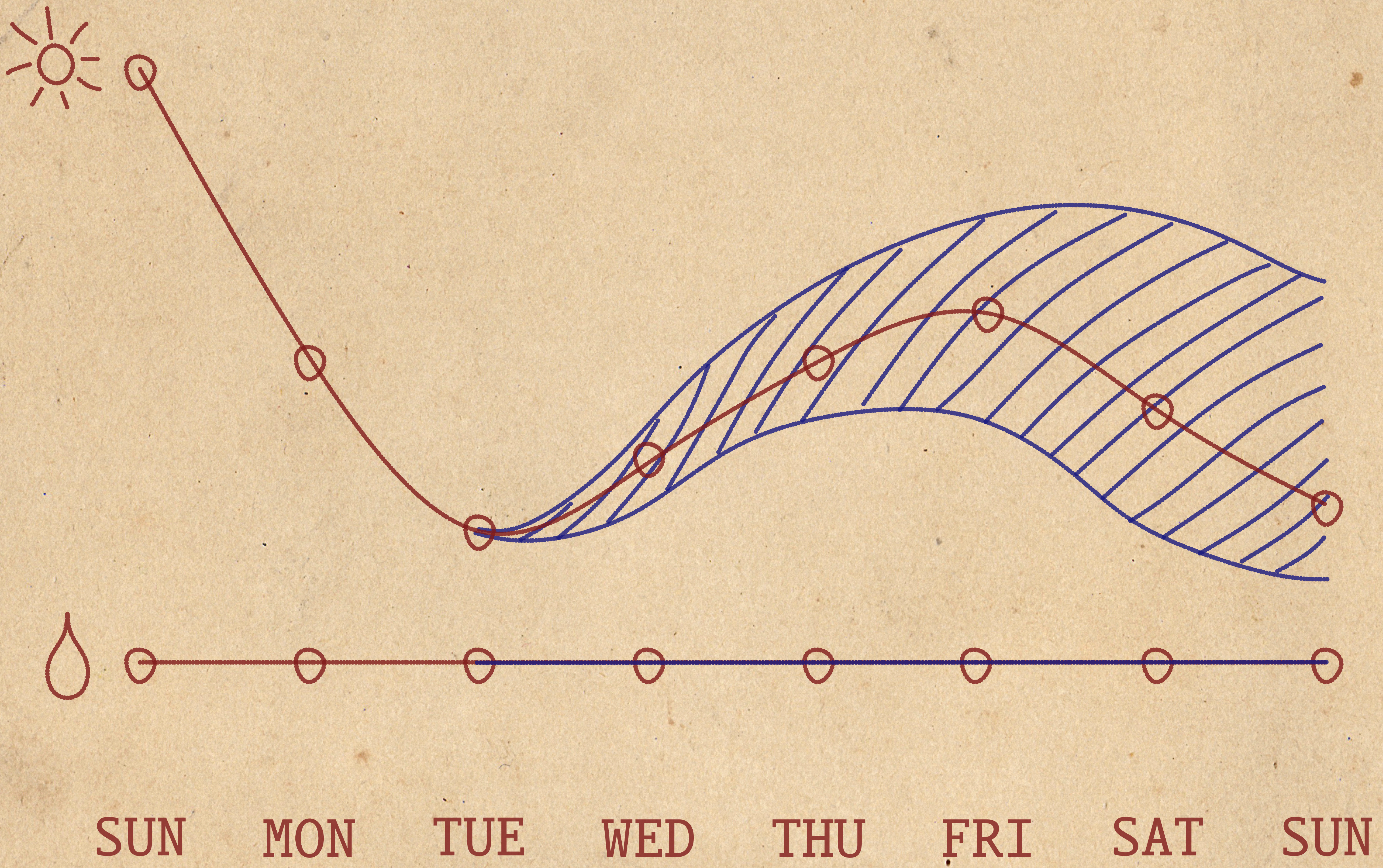

GRAPH DRAWING

Consider a typical graph drawing scenario.

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Input: a graph

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G = ({1,2,3,4}, {(1,2),(1,3),(2,3),(2,4)})

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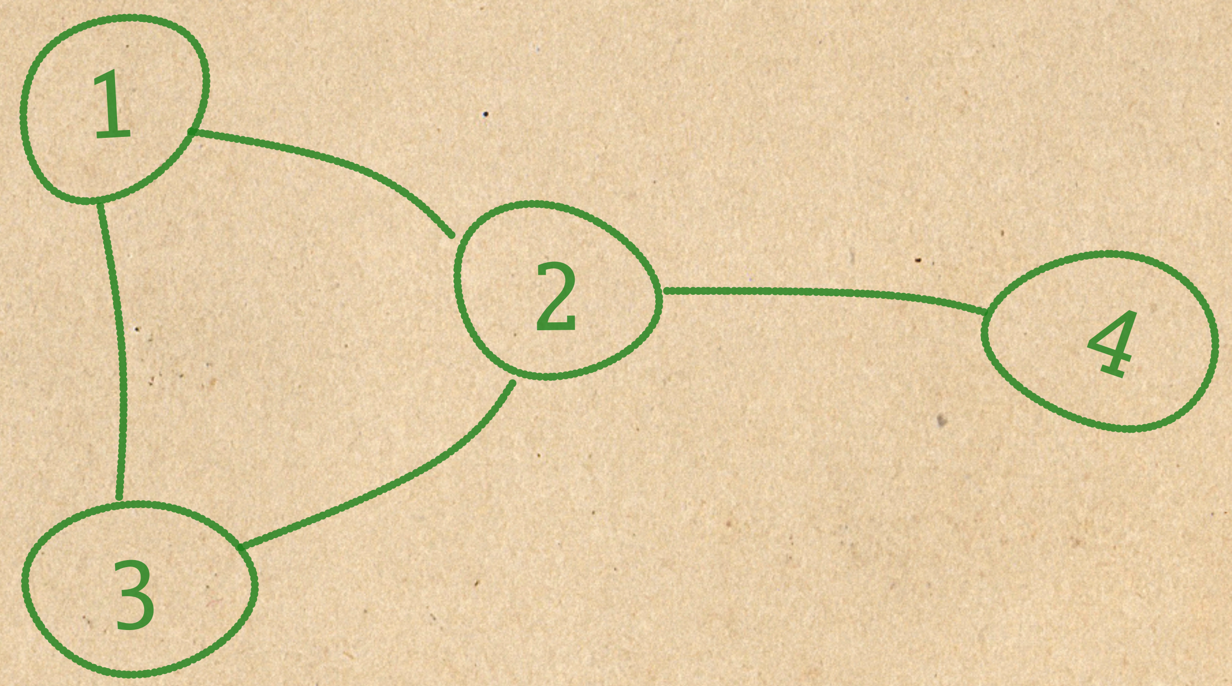

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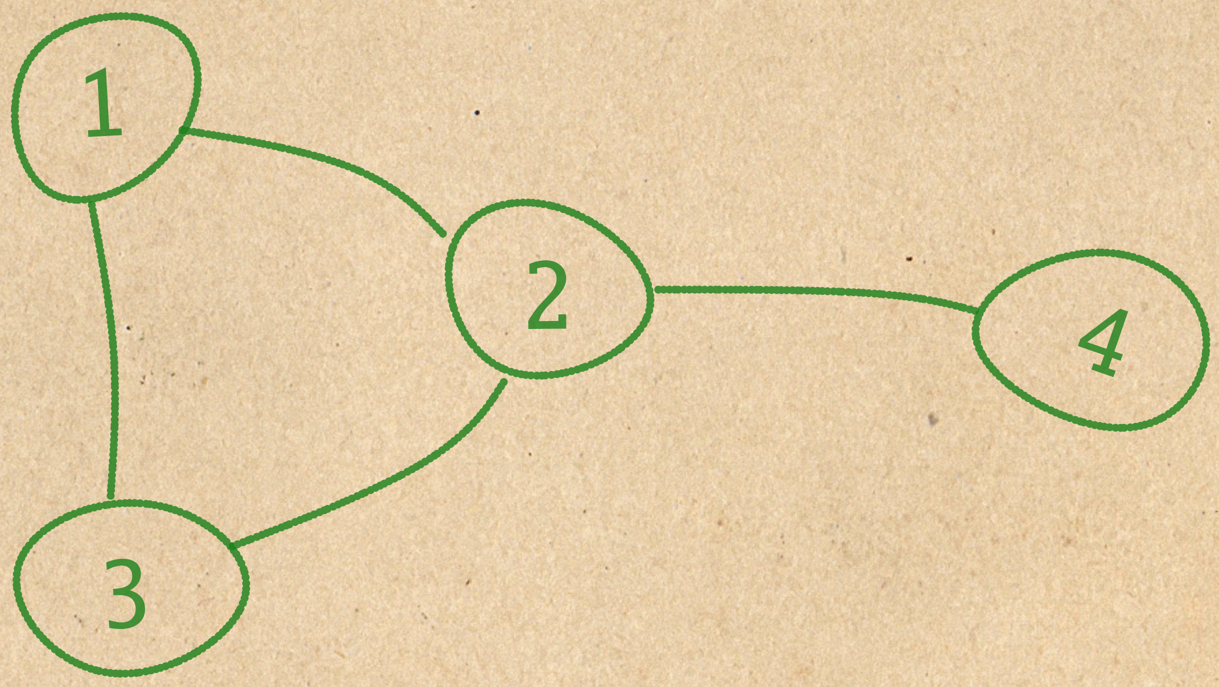

We need to find out where the vertices are!

Some graphs are *non-geometric*.

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Vertices have no intrinsic geometric meaning.

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Other graphs are *geometric*.

Vertices already have a location!

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They serve to illustrate the graph structure.

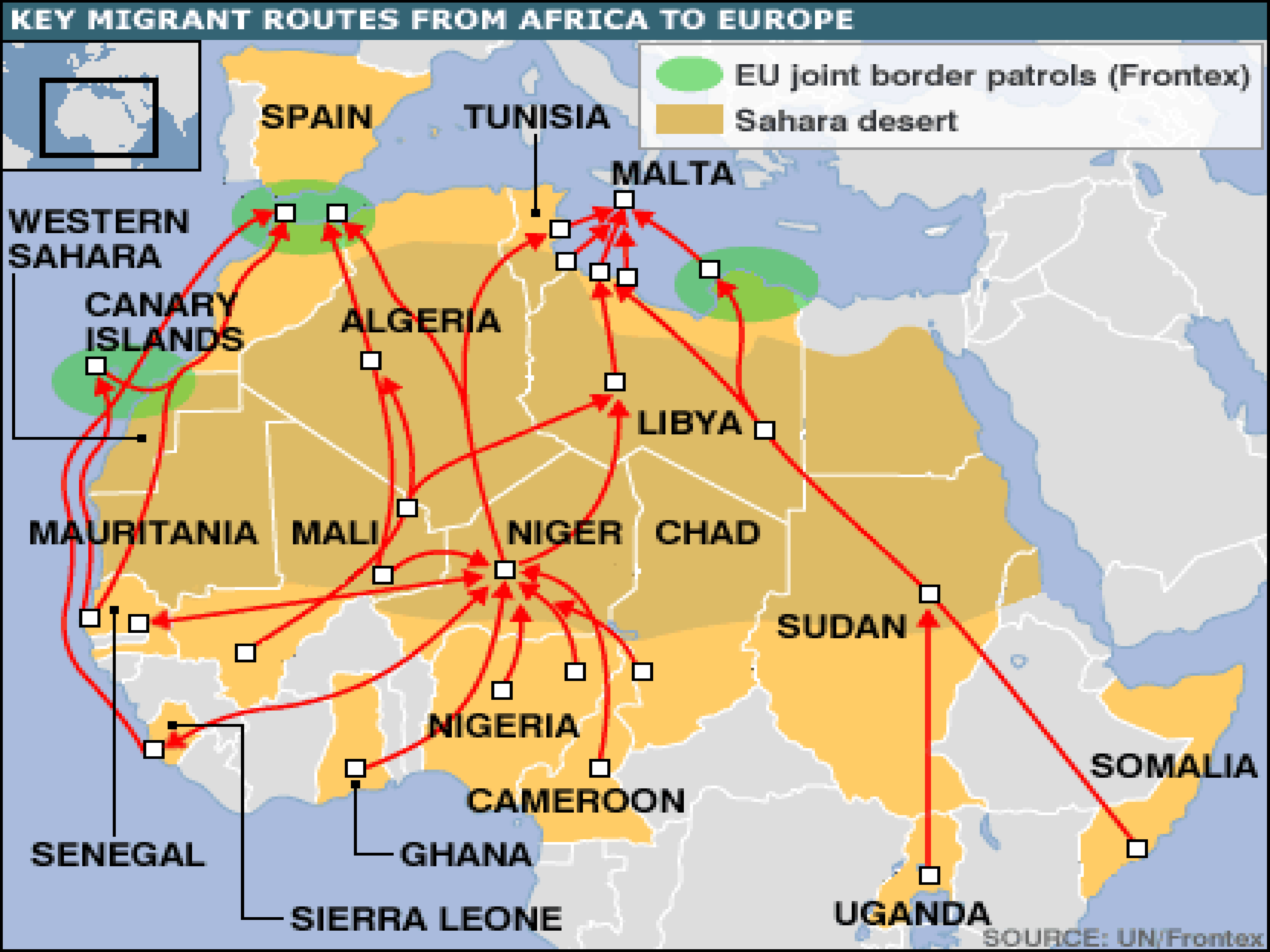
These graphs are no fun.

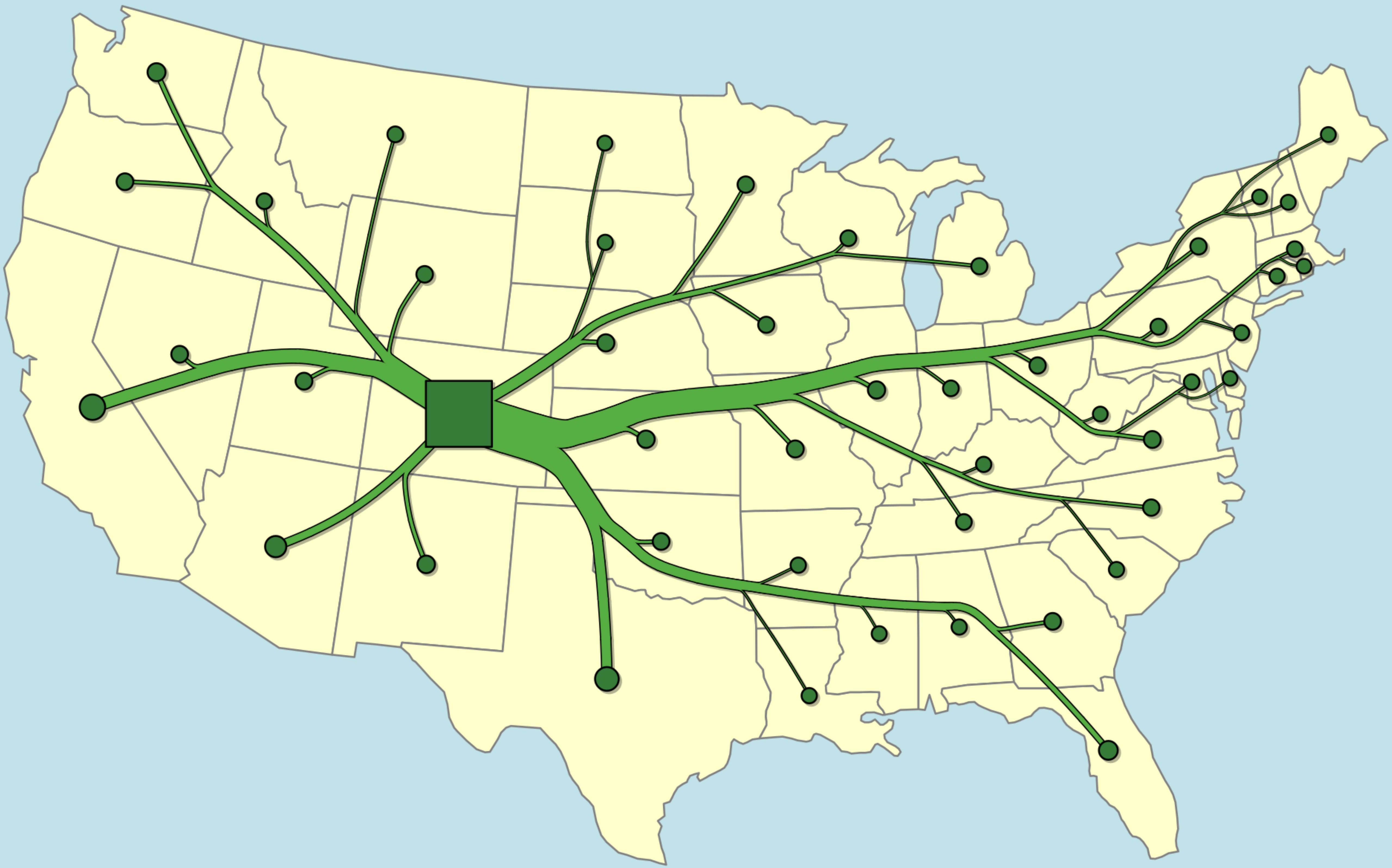
Other graphs are *geometric*.

Vertices already have a location!

But then, what is left to be drawn?


Lancaster
Sylmar (Hubbard/Borden)
Mission College
San Fernando
Maclay
Pacoima
Arleta
Panorama City
Sun Valley
Burbank Media Ctr
Sonora
Alameda
GLENOAKS
Glenoaks
Brand Broadway
Chevy Chase
Oxnard
Chatsworth
Northridge
Burbank Airport
Van Nuys
Sherman Way
Victory
Burbank
Canoga Park
Warner Center
DeSoto
Reseda
Balboa Park
Van Nuys Civic Ctr
Winnetka
White Oak
Sepulveda
Valley Plaza
Glendale
Eagle Rock
Figueroa
Verdugo Road
Sierra Madre Villa
Lake
Monrovia
Irwindale
Glendora
La Verne
North Pomona
Montclair
San Bernardino
Allen
Arcadia
Duarte
Azusa
San Dimas
Claremont
Memorial Park
Del Mar
Fillmore
Mission (South Pasadena)
Highland Park
Heritage Square/ Arroyo
Southwest Museum
Lincoln Heights/ Cypress Park
Chinatown
North Hollywood
Universal City
Sherman Oaks
Griffith Park
Franklin Hills
Atwater Village
Rowena
Edendale
Echo Park
Dodger Stadium
Fort Moore Hill
Plaza Olvera
Covina
Baldwin Park
San Gabriel Mission
Fremont
Rosemead
Hillside Village
Garfield
San Gabriel Bl
Baldwin
El Monte
Lincoln Park/ LAC+USC Med Ctr
Cal State LA
Santa Anita Av (BUSWAY)
Pomona
Riverside
Hollywood/Highland
Hollywood/Vine
Hollywood/Western
Vermont/ Sunset
Sunset Junction
Melrose Hill
Arclight
Billy Wilder Square
Silver Lake
Union Station
Getty Center
Crescent Heights
Vermont/Sta Monica LA City College
Music Center
Civic Center
UCLA
West Hollywood
Vermont/Beverly
Temple
5th/Flower
Pico Aliso
Soto
Maravilla
Pomona/Beverly
Industry
Montebello
Mariachi Plaza
Lorena
East LA Civic Ctr
Little Tokyo
Century City
Beverly Center
Miracle Mile
Wilshire/ Western
Wilshire/ Vermont
7th/Metro
Pershing Square
Brentwood
WILSHIRE
Pacific Palisades
Allenford
Wilshire/ Veteran
Rodeo Drive
Robertson
Wilshire/ La Cienega
Park Mile
Wilshire/ Crenshaw
Wilshire/ Normandie
Westlake/ McArthur Park
6th/Main
Pico
Roppongi
Whittier/Soto
Whittier/Lorena
Whittier/Atlantic
Pico Rivera
West Whittier
Whittier Central
Colima
Beach Bl
La Habra
Garibaldi
Veterans' Administration
La Cienega/Pico
Adams
La Cienega/Cadillac
Dorsey
Jefferson Park
Dockweiler
Wilshire/ 26th St
Motor
Rancho Park
Bundy
20th St
Lincoln
Culver Junction
La Cienega
Foshay
USC
Exposition Park
Grand
San Pedro
Wilshire/ 14th St
Culver City Center
Vermont Village (Vermont/Vernon)
Washington
Vernon
Fox Hills
Chesterfield Square (Western/Slauson)
Hyde Park
Slauson
Santa Monica Place
Ocean Park
Venice
Marina Del Rey
Manchester/ Sepulveda
Florence
Inglewood
Commerce
Firestone
La Tijera
Westchester
103rd St/Kenneth Hahn
Terminals 1-3
Manchester Square
Hawthorne
Vermont
Avalon
Lynwood
Lakewood
Norwalk
Santa Fe Springs
Int'l Terminal and Terminal 4
LAX
Aviation
Crenshaw
Harbor Freeway
Rosa Parks
Paramount
Hoxie
Buena Park
Terminals 5-7
Mariposa/Nash
El Segundo/Nash
Compton
Bellflower
Gahr
Douglas/Rosecrans
Artesia
Artesia
Fullerton
Riverside
PACIFIC OCEAN
Marine/Redondo Beach
Del Amo
Cypress/La Palma
Anaheim
Anaheim Canyon
Manhattan Village
Cypress College
Manhattan Beach
South Bay Galleria
Wardlow
Stanton
Willow
Torrance
Garden Grove
Pacific Coast Highway
Fairview
Anaheim
Orange
Main
Harbor City
5th Street
Santa Ana
Pacific
1st Street
Santa Ana College
Ross
Oceanside
Transit Mall


KEY MIGRANT ROUTES FROM AFRICA TO EUROPE
EU joint border patrols (Frontex)
Sahara desert
SPAIN
TUNISIA
MALTA
WESTERN SAHARA
CANARY ISLANDS
ALGERIA
LIBYA
MAURITANIA
MALI
NIGER
CHAD
SUDAN
NIGERIA
CAMEROON
SOMALIA
SENEGAL
GHANA
SIERRA LEONE
UGANDA
SOURCE: UN/Frontex



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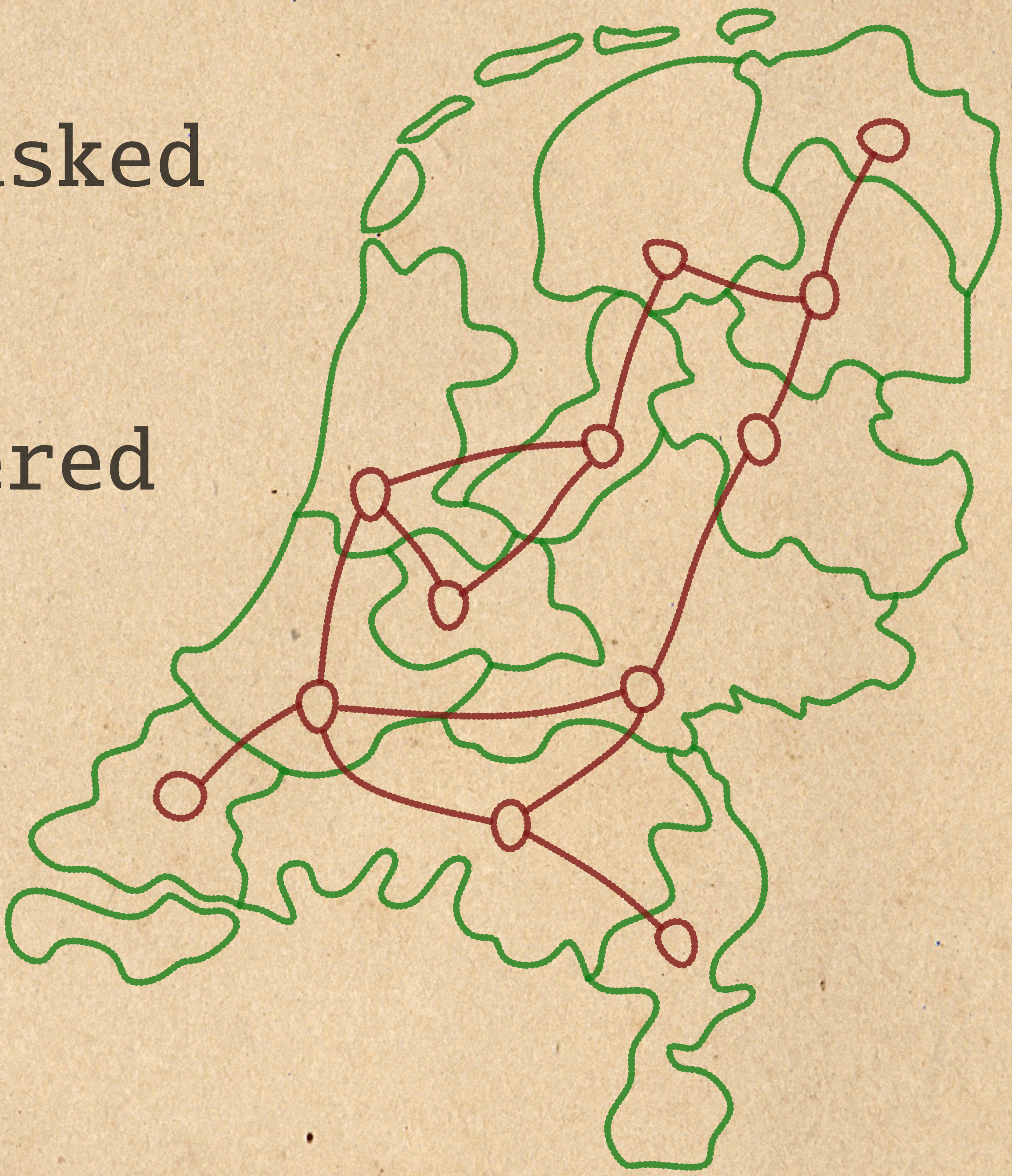

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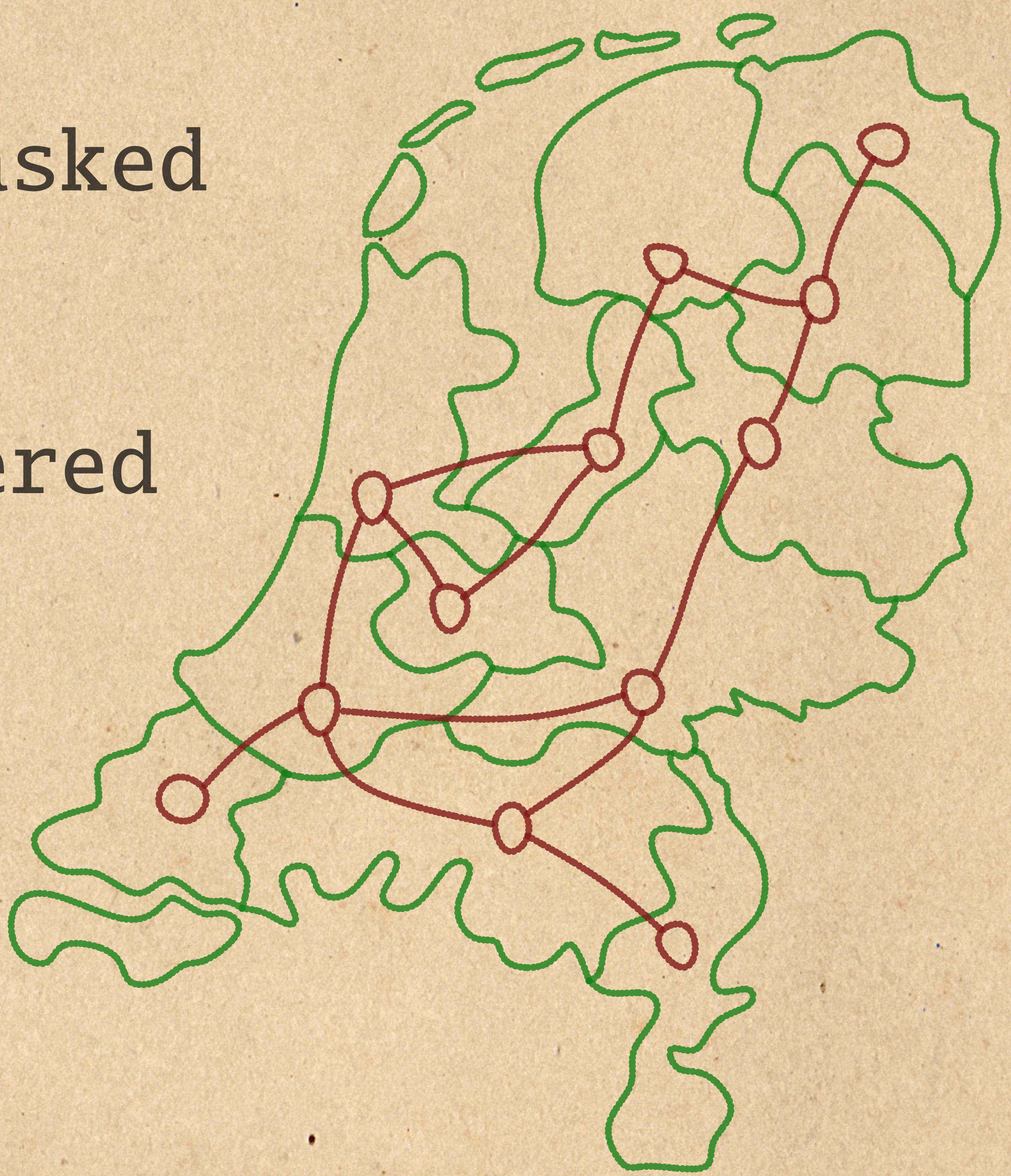

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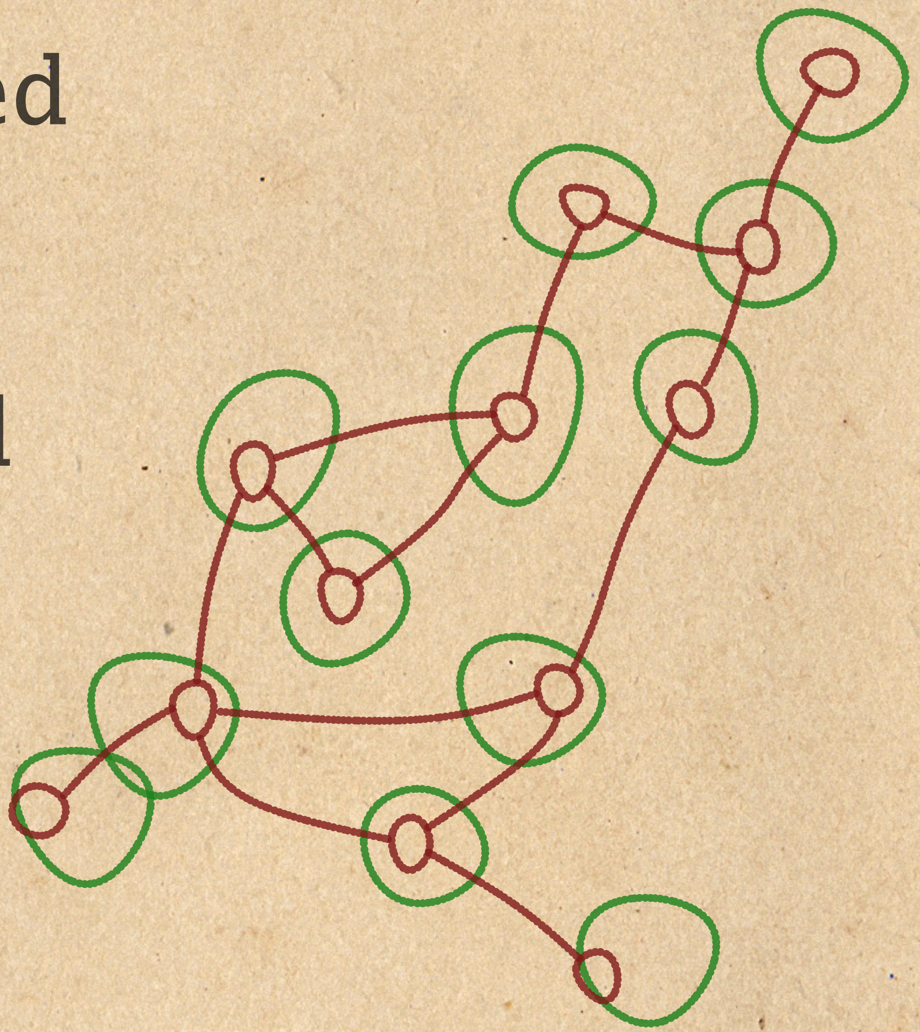

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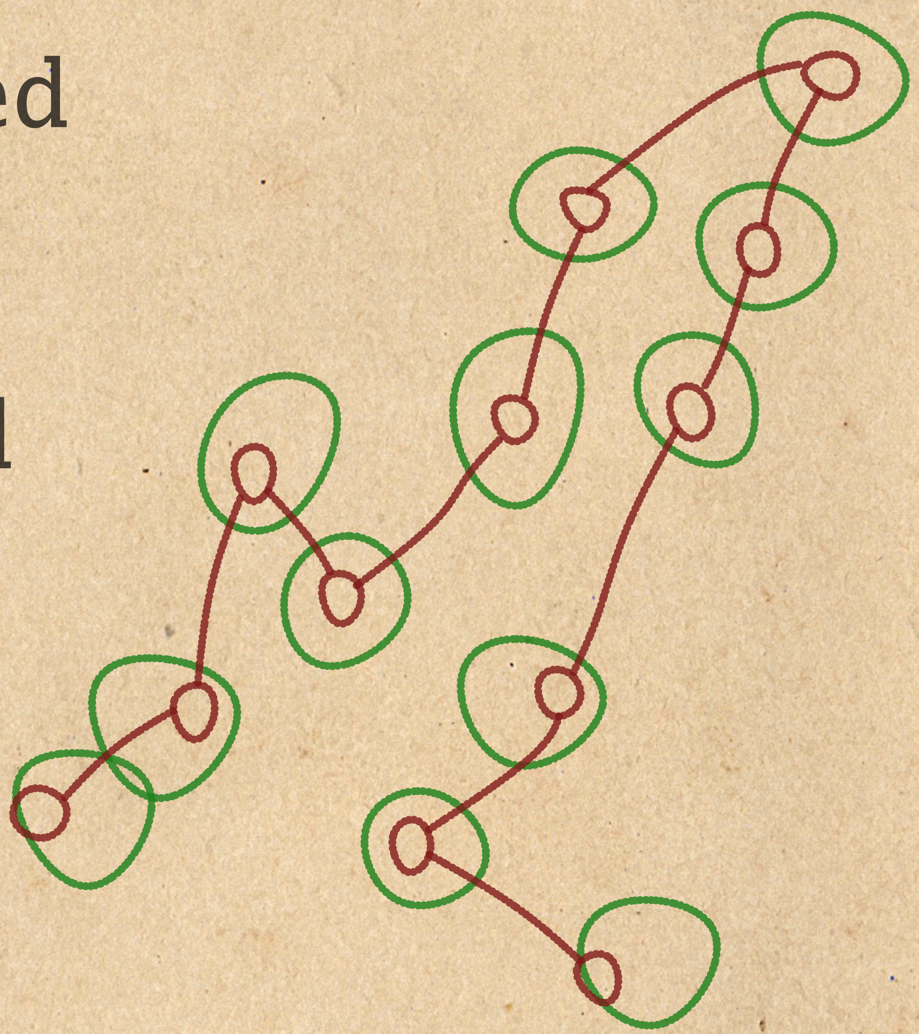

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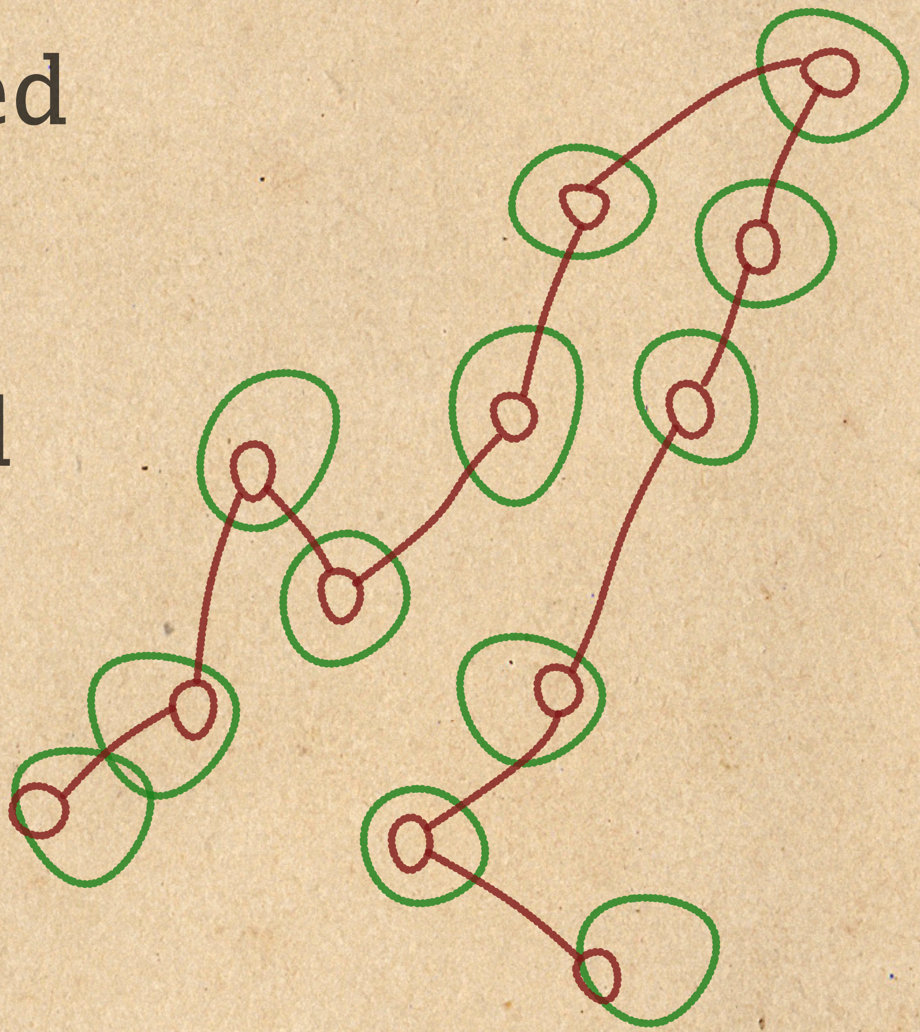

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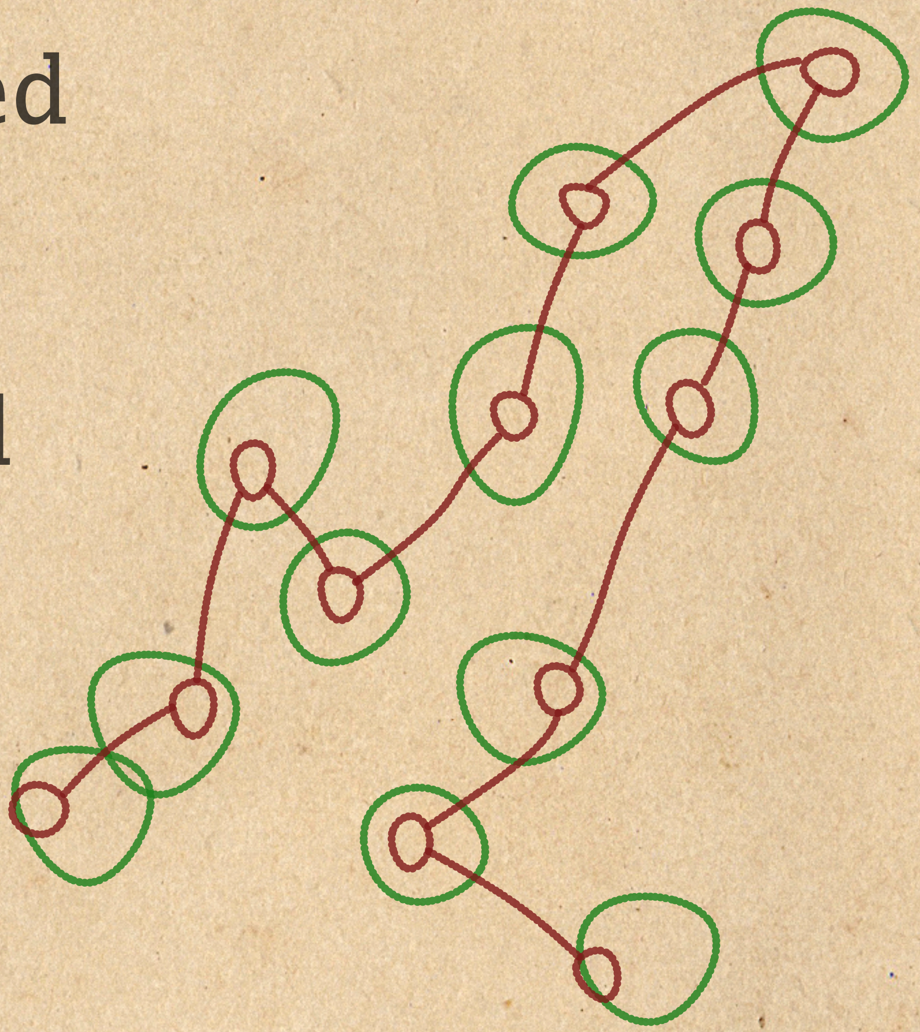

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GEOMETRIC UNCERTAINTY

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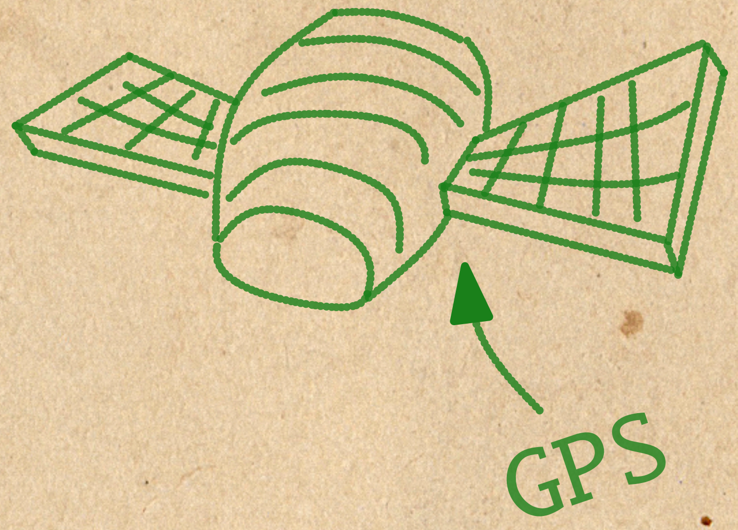

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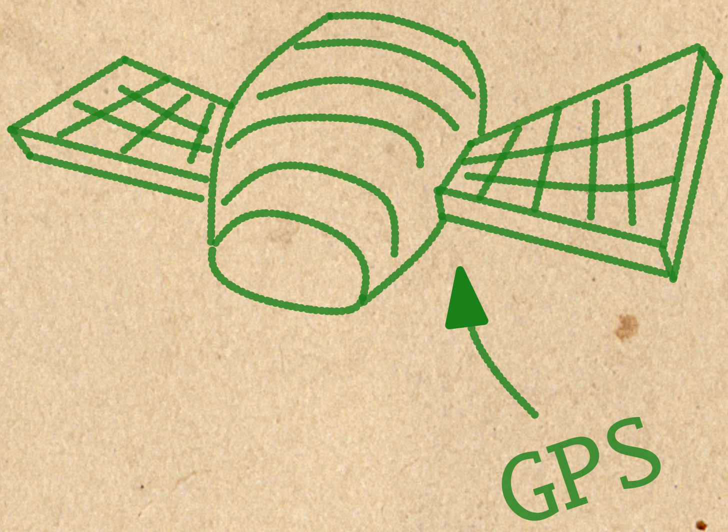

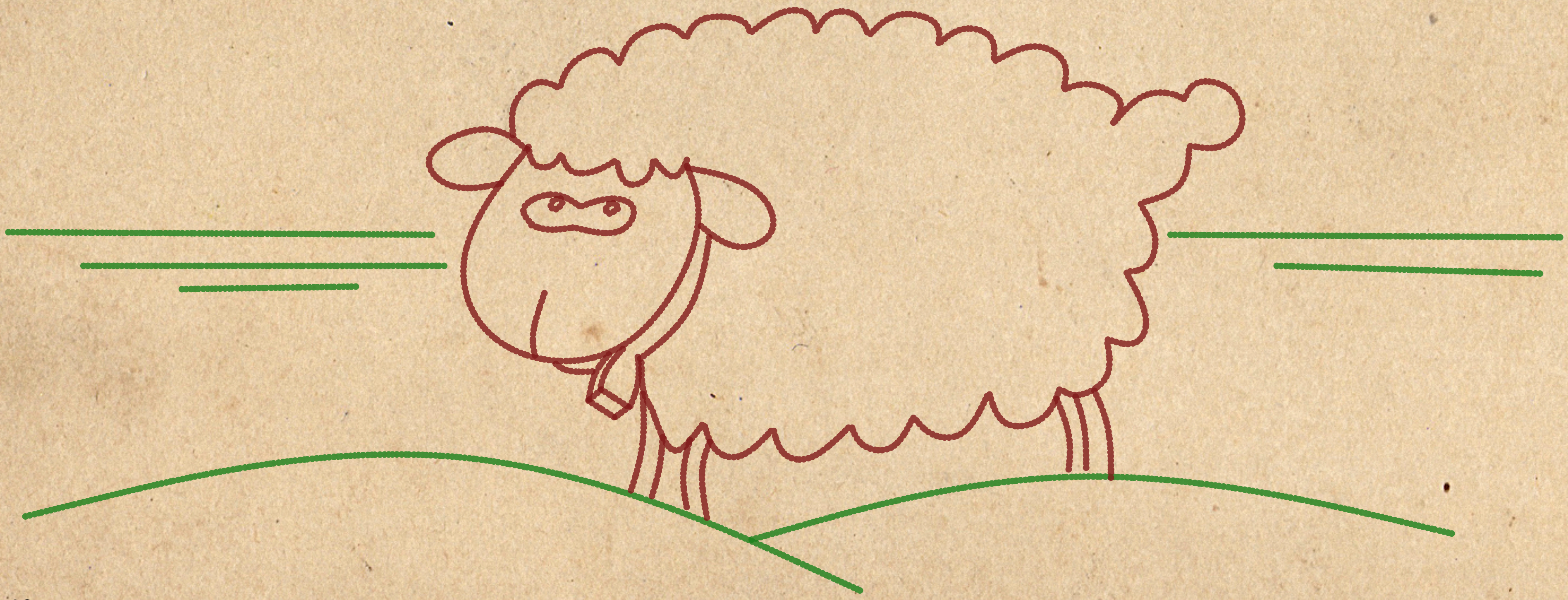

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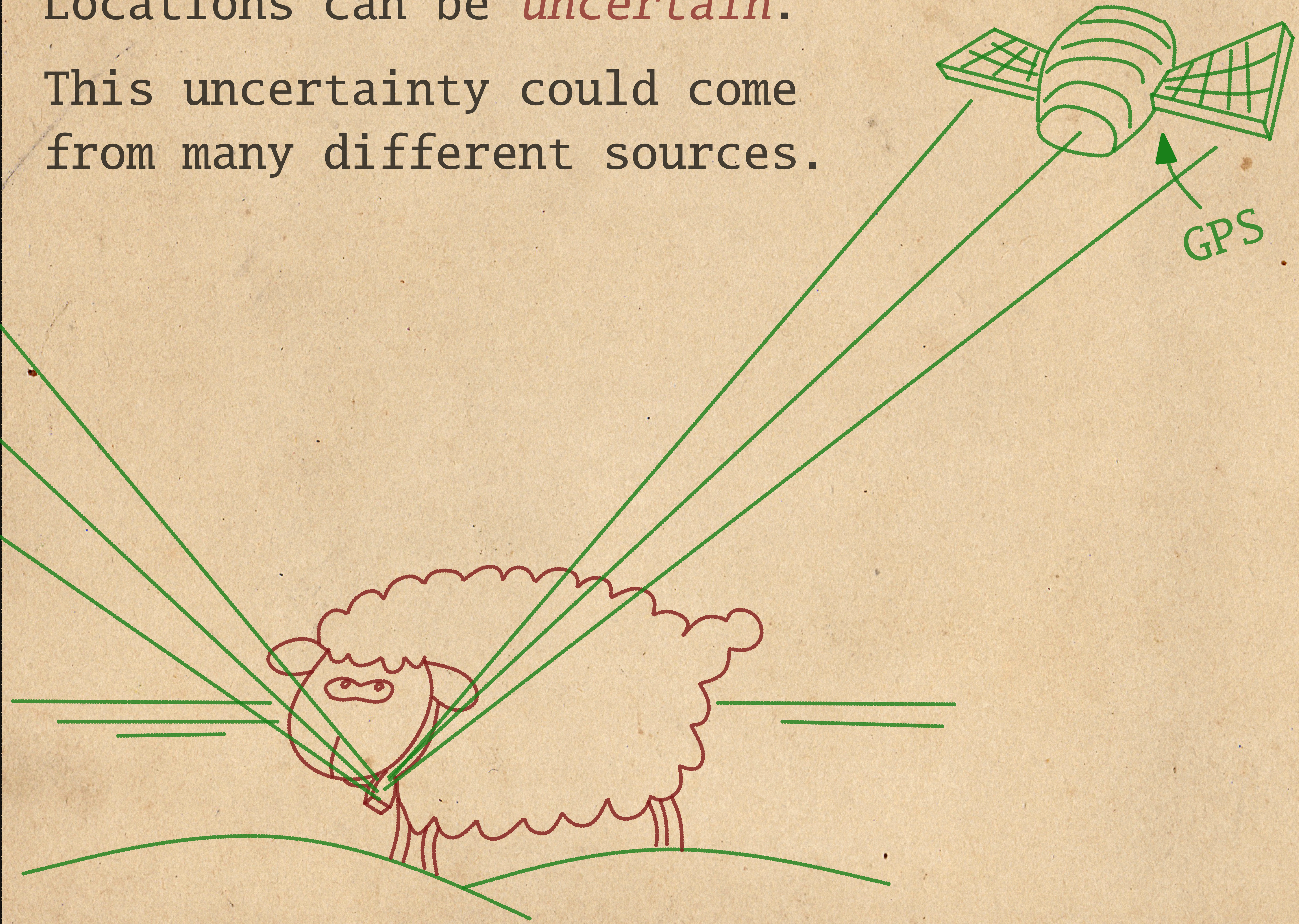

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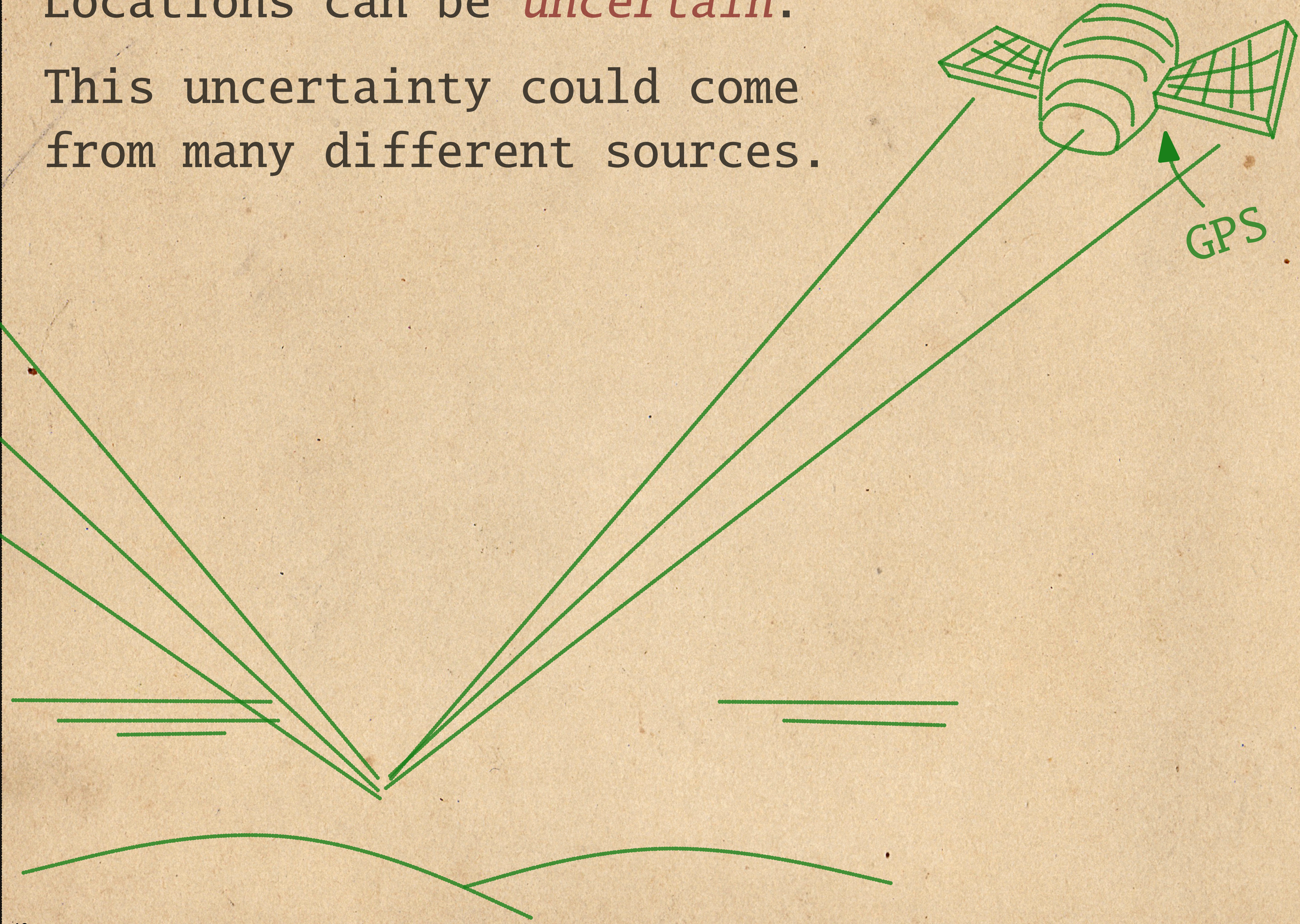

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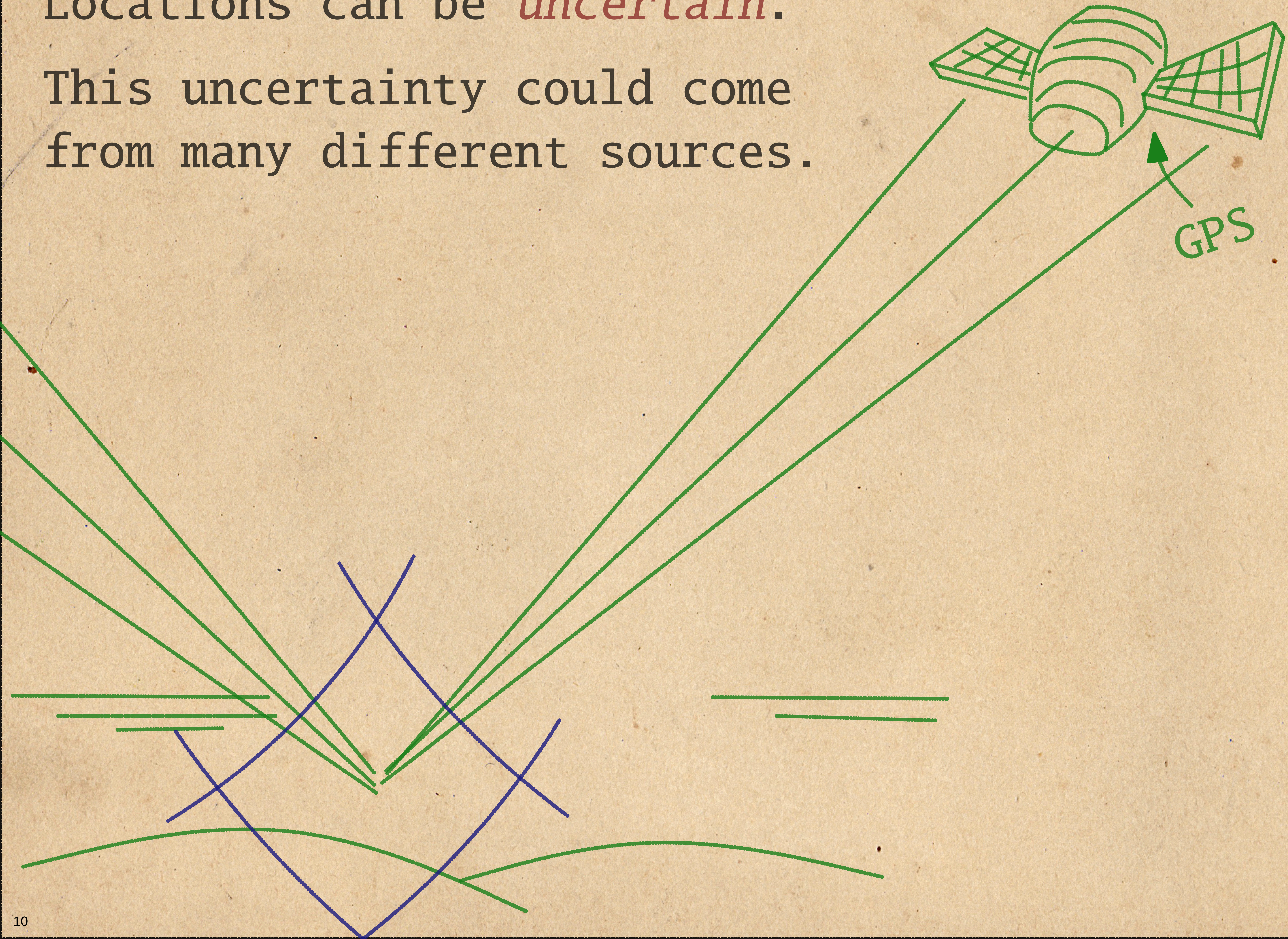

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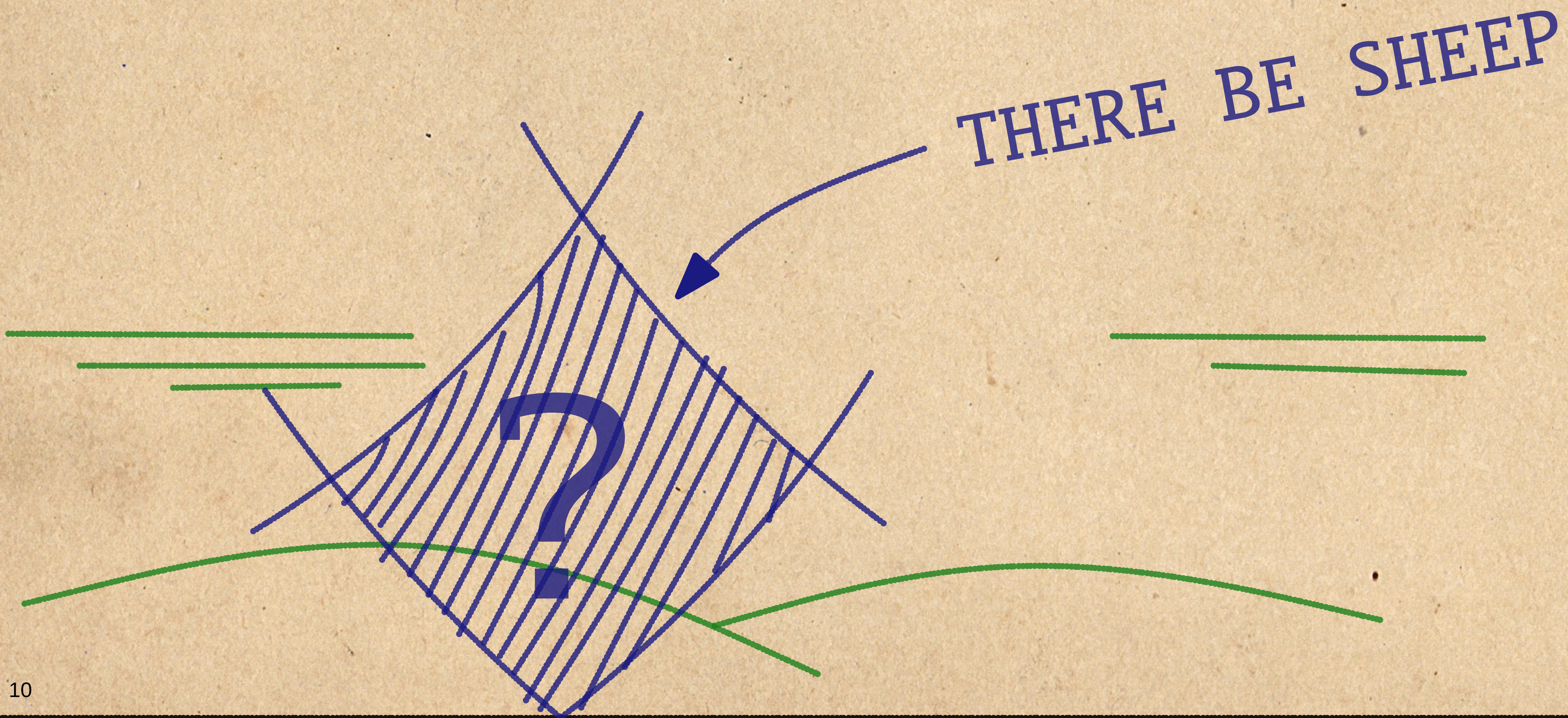

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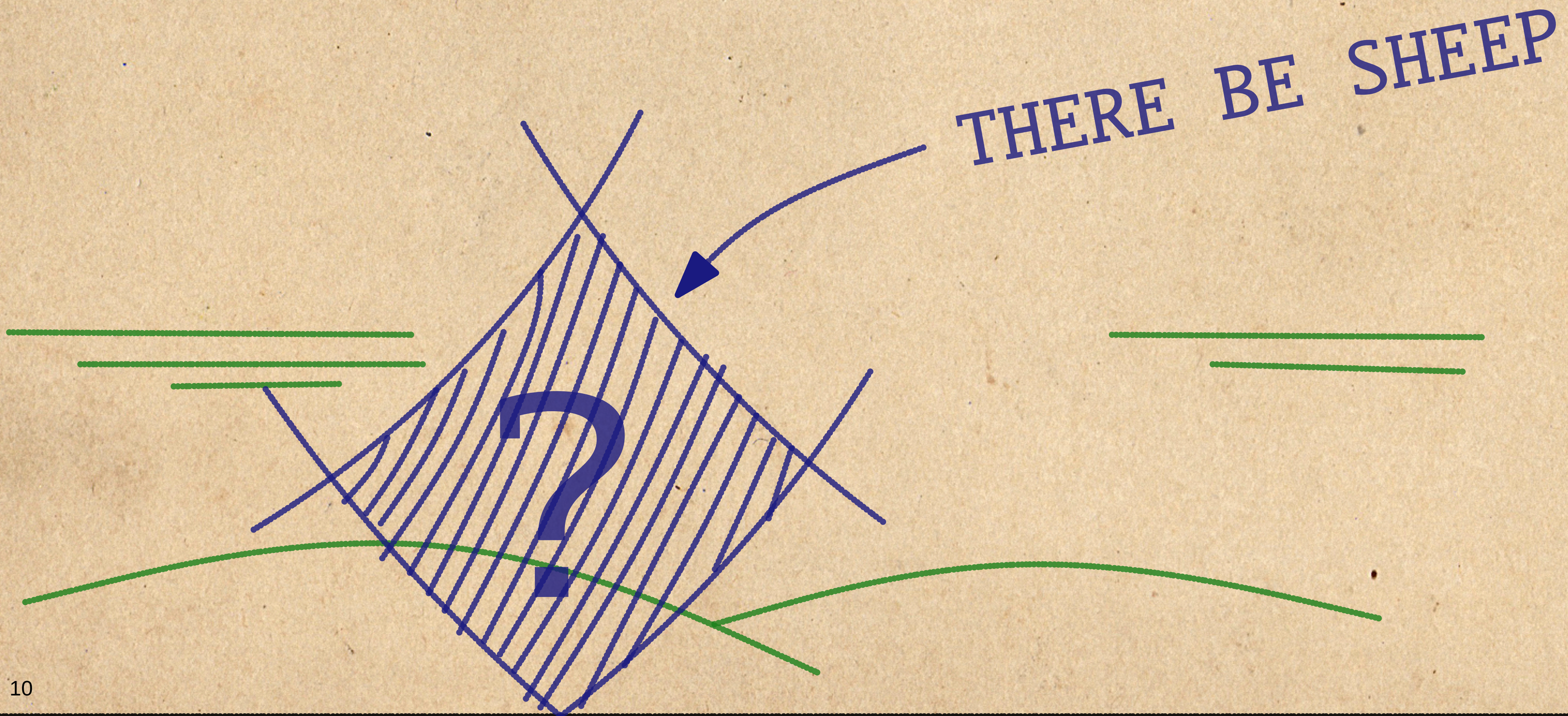

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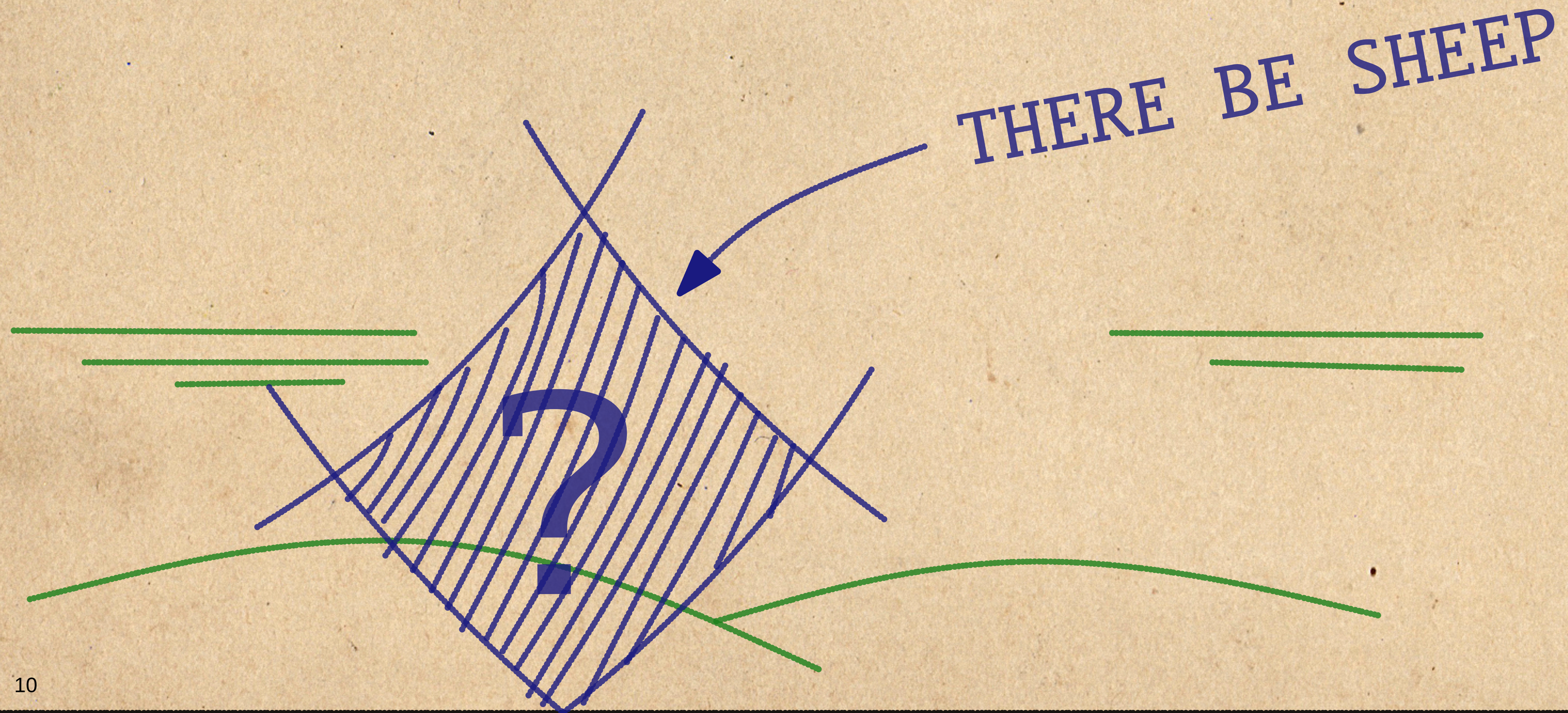

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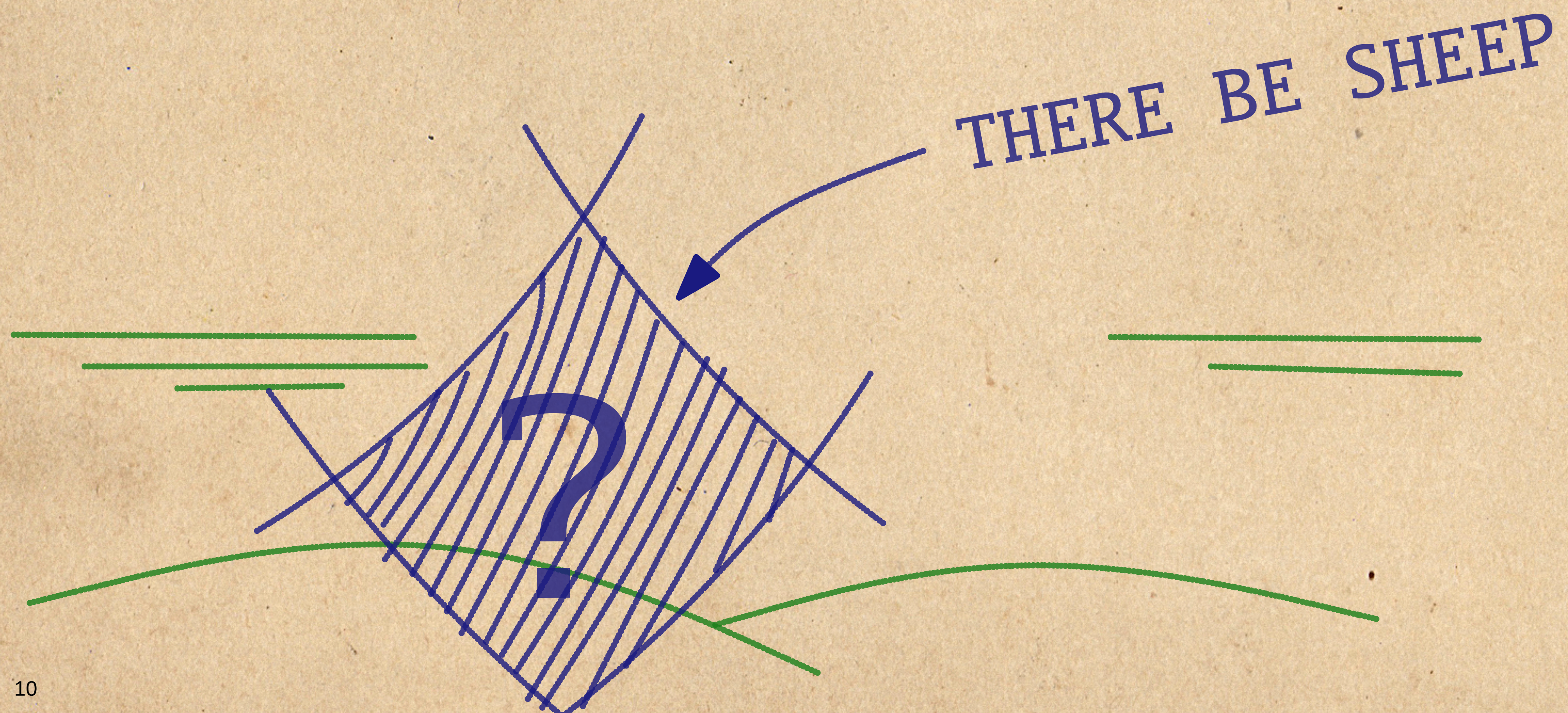

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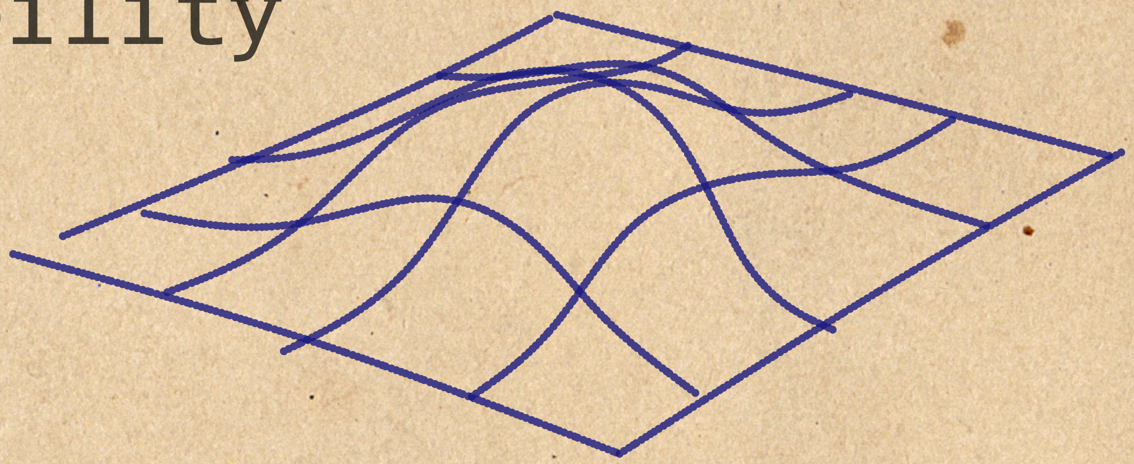

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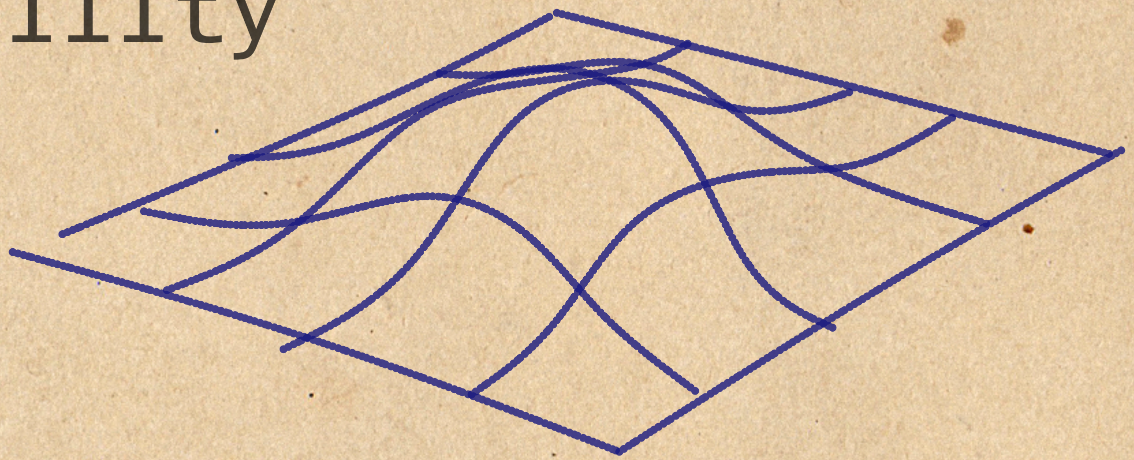

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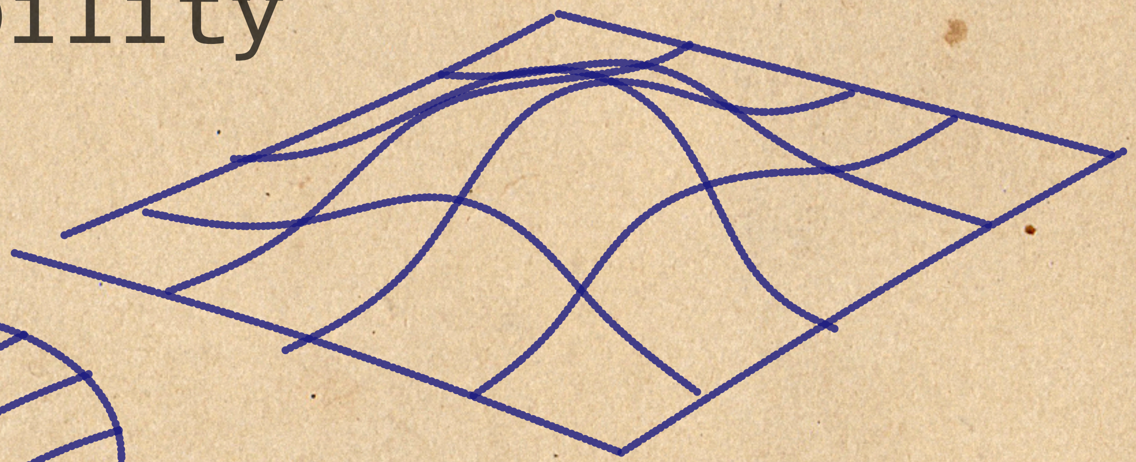

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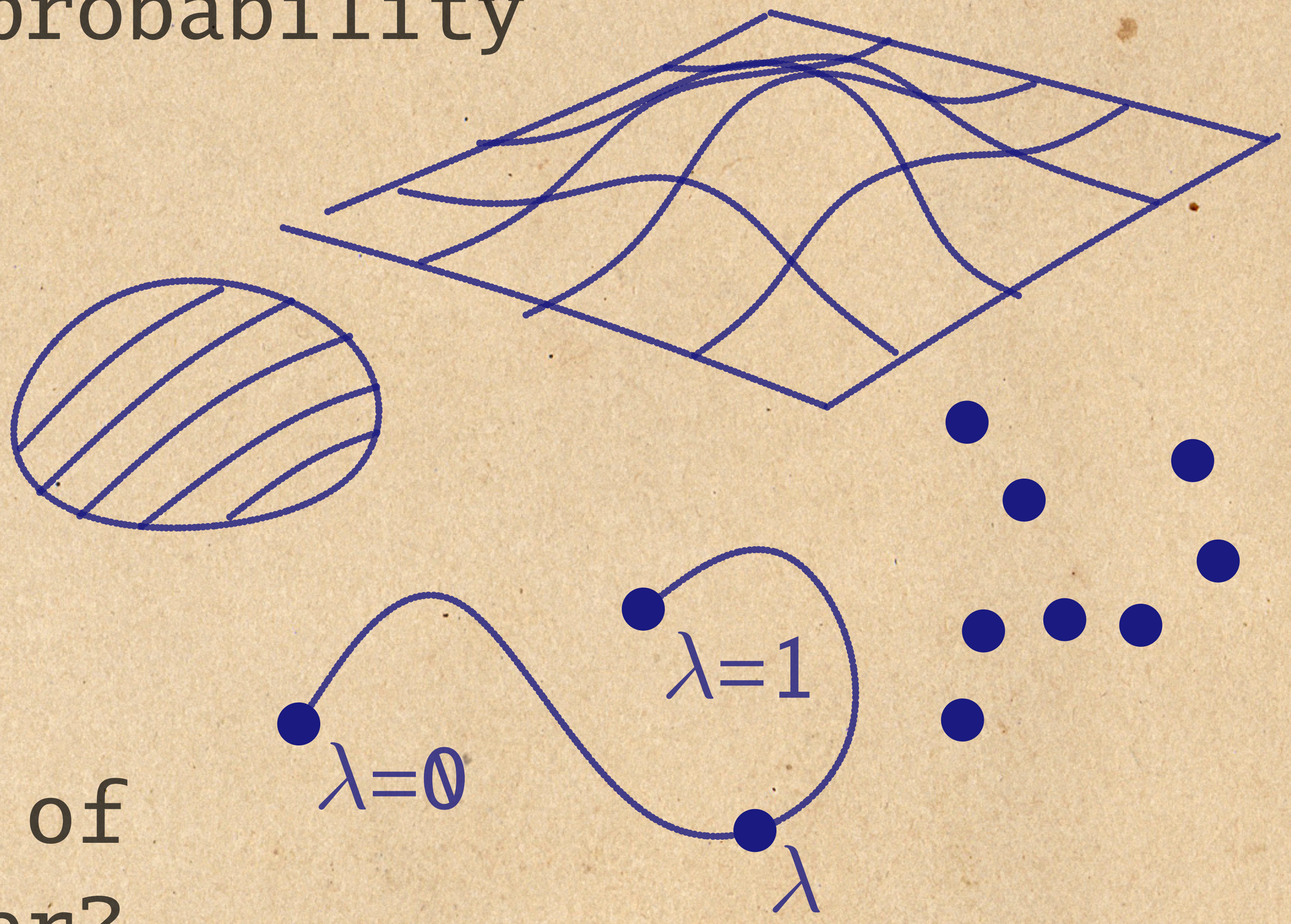

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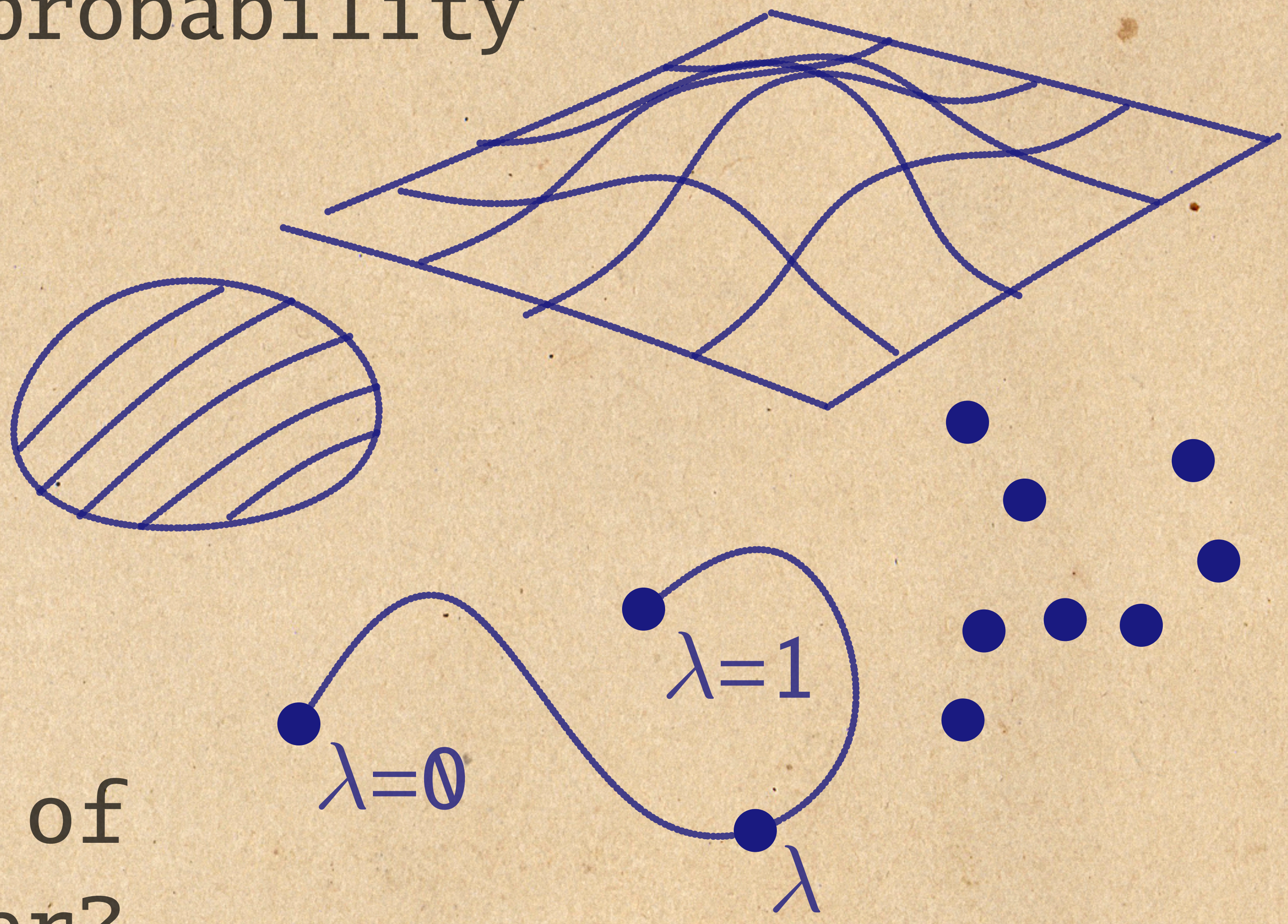
Etc...



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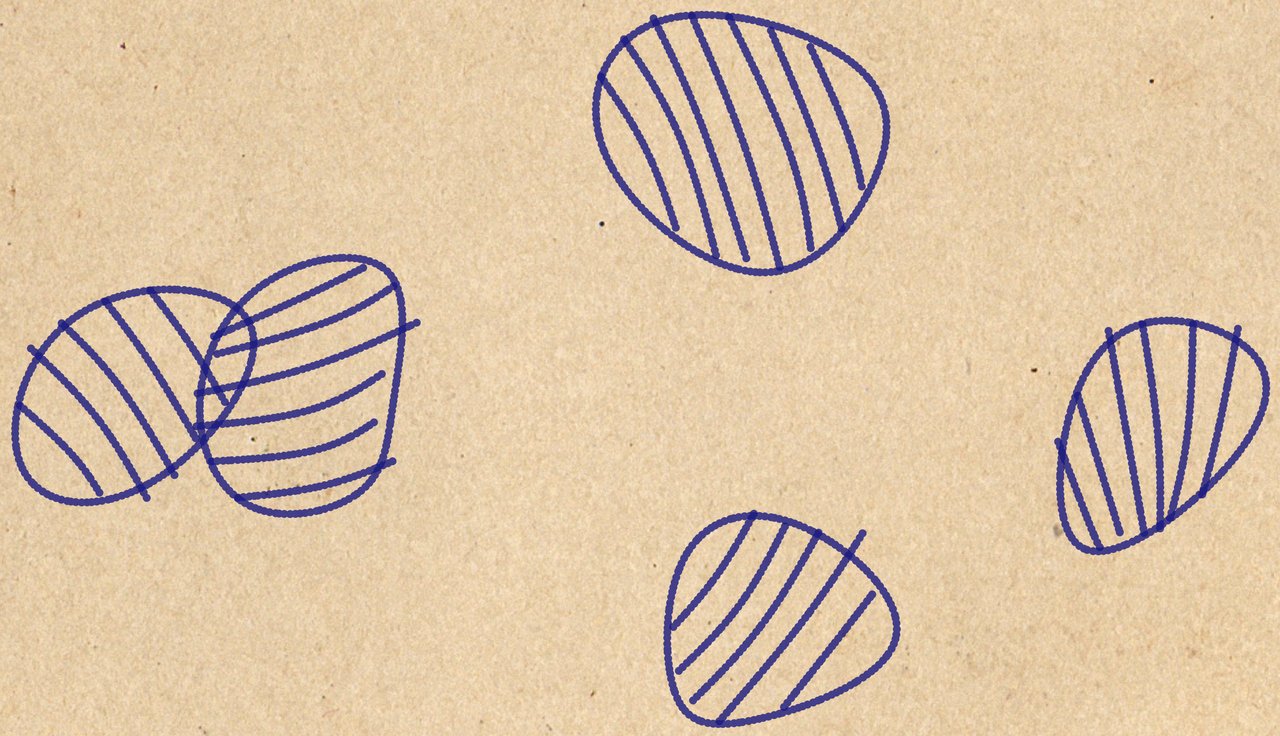

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When the input to a geometric problem is uncertain...

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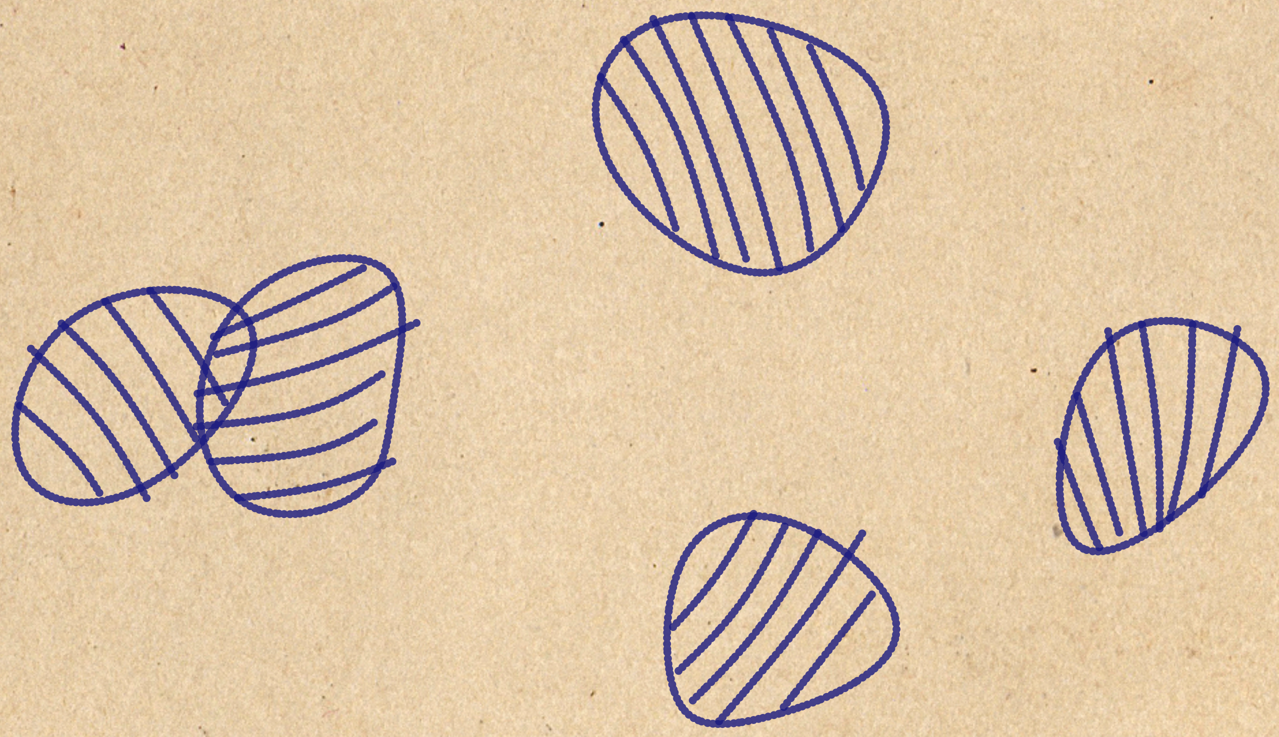

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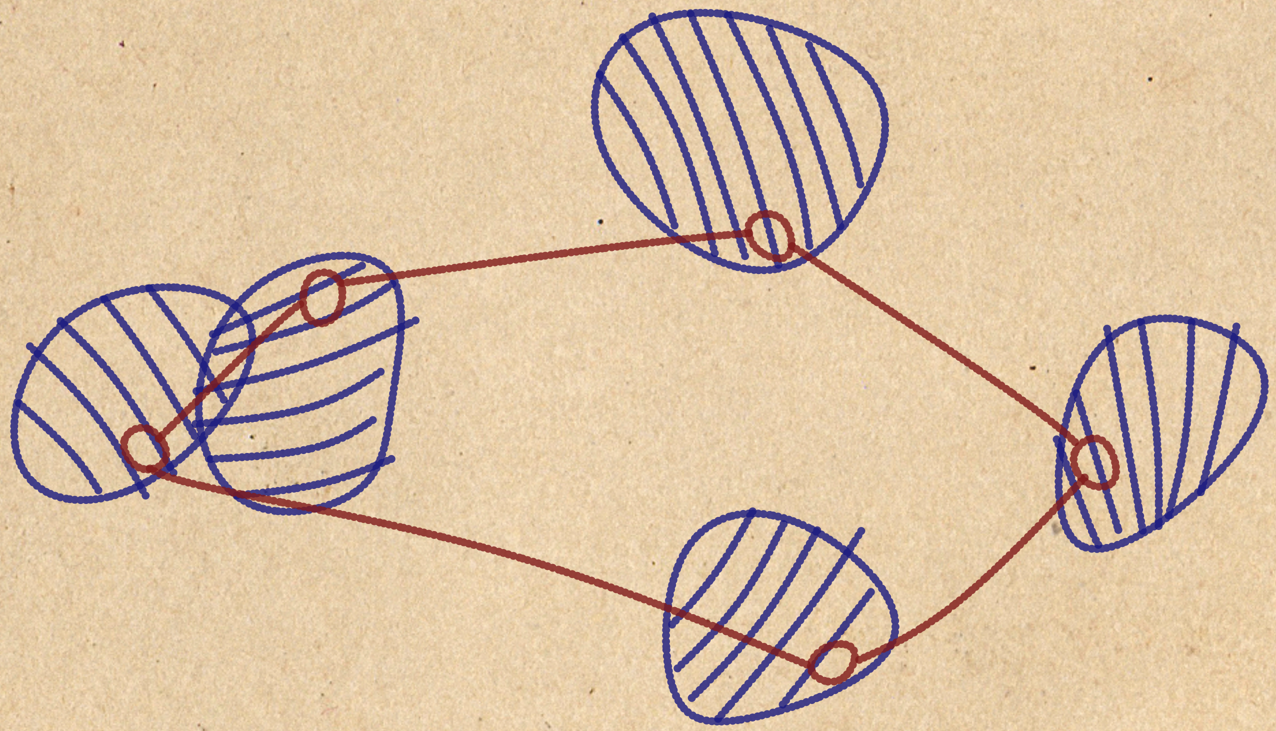

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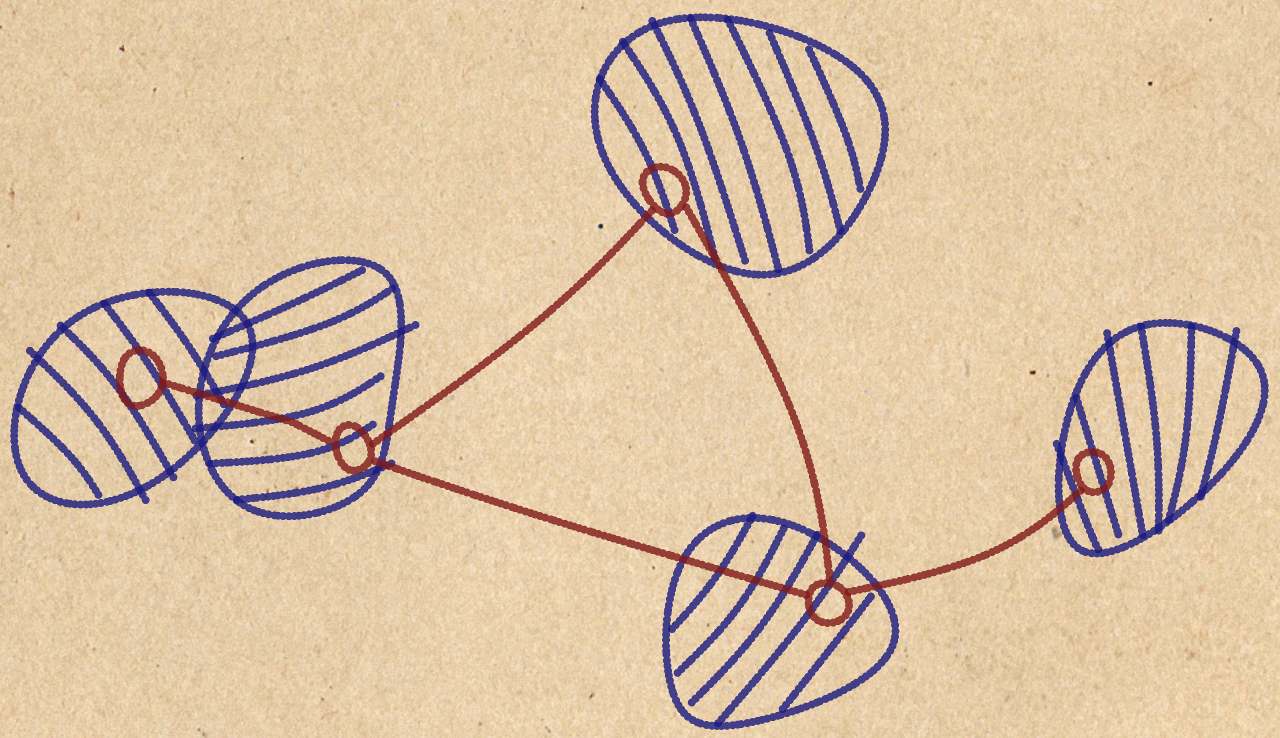

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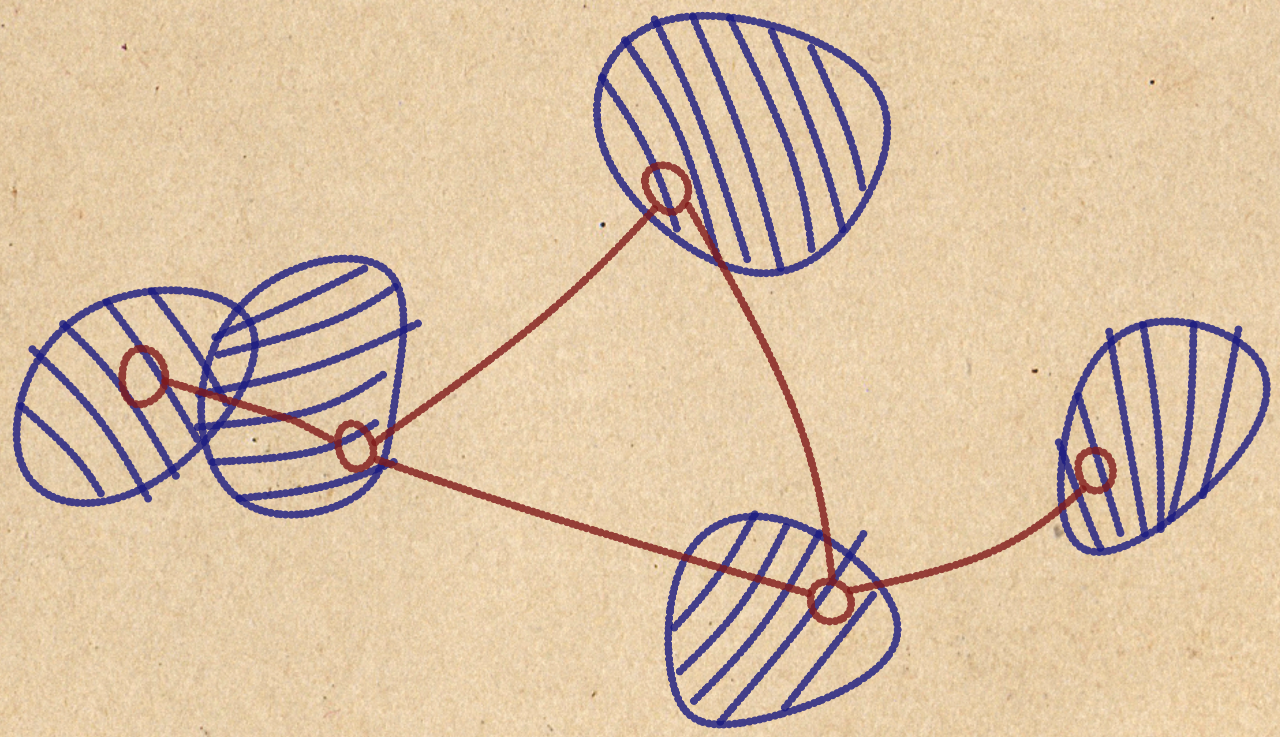

How do we capture/store/describe/visualise this output uncertainty?

Today, let's focus on *visualising* geometric uncertainty.

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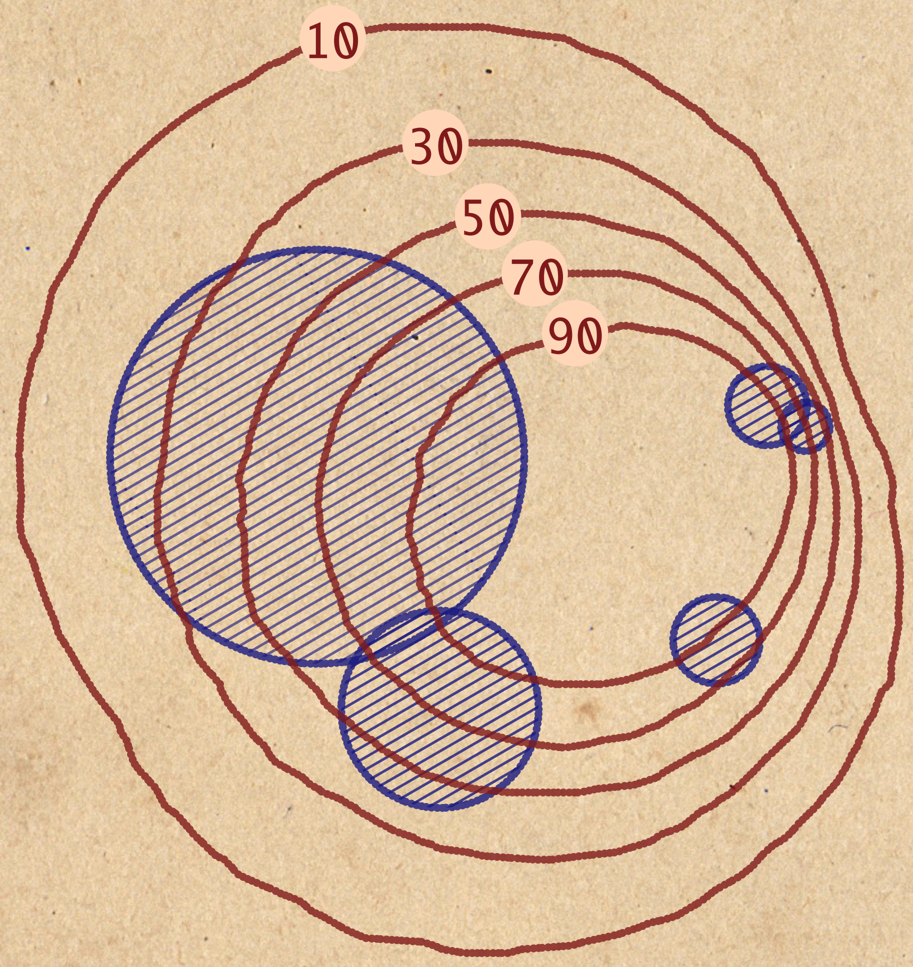

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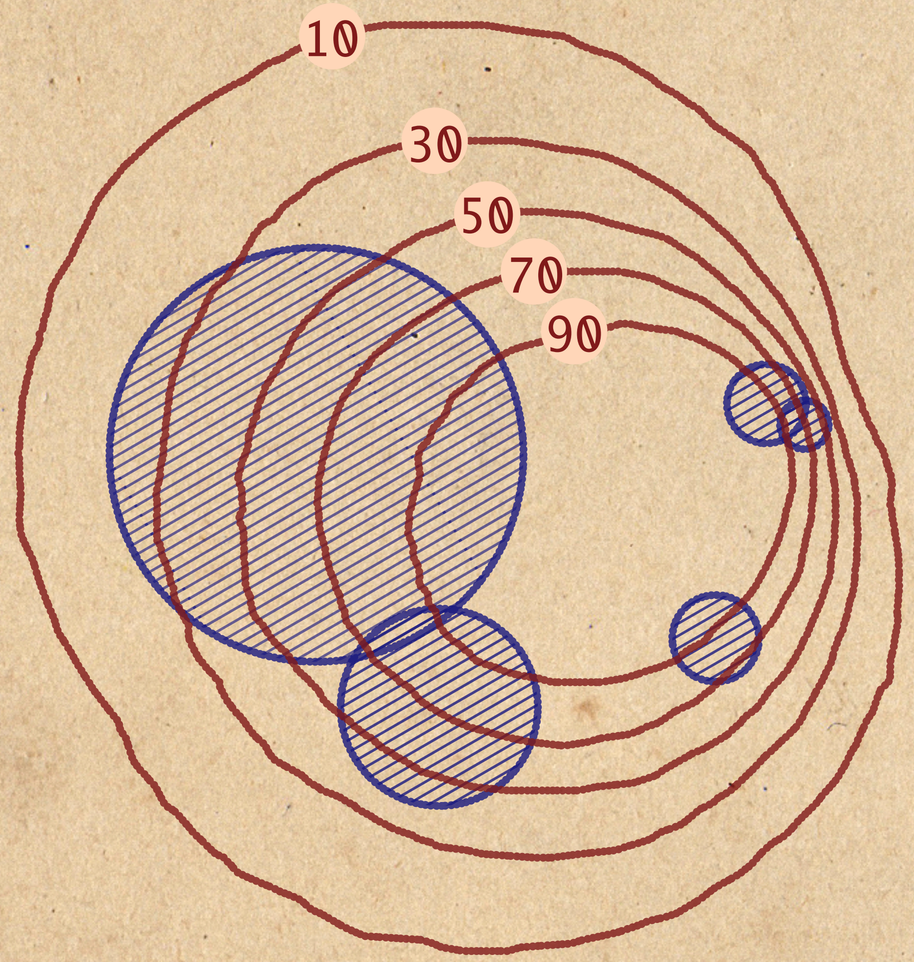

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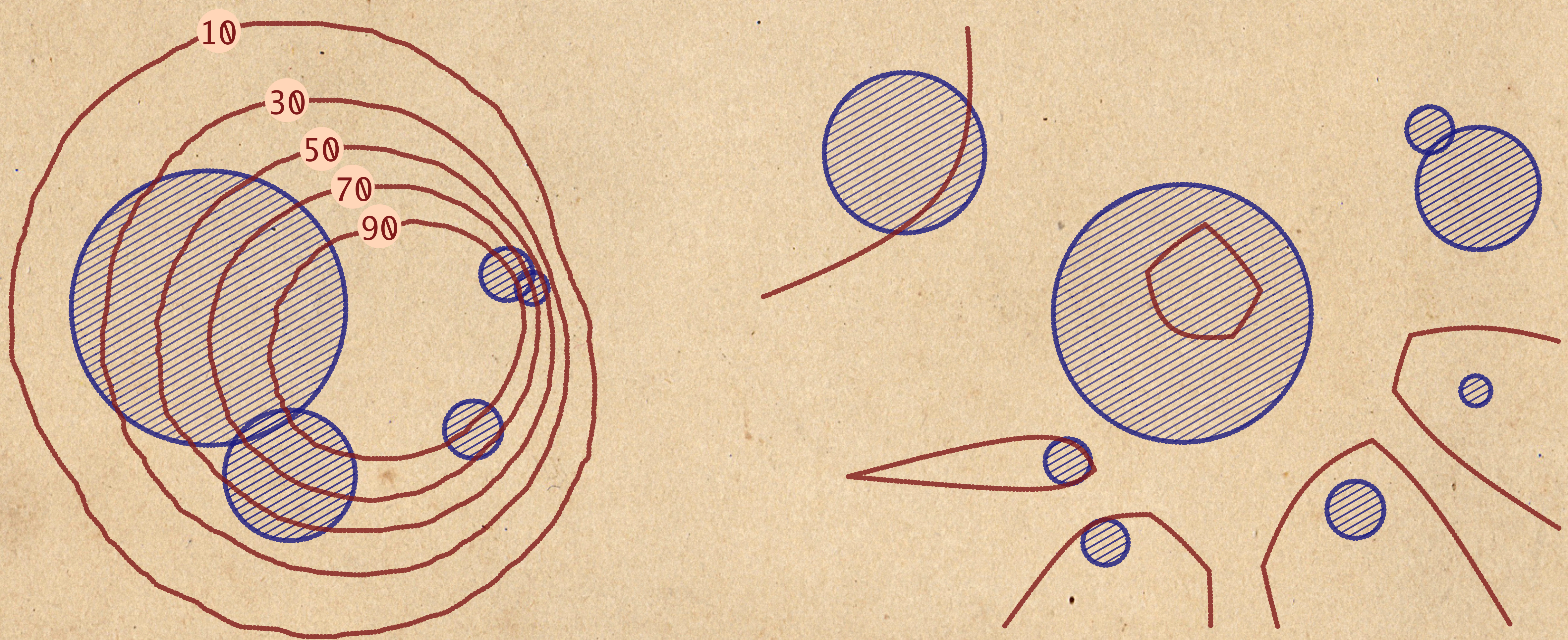

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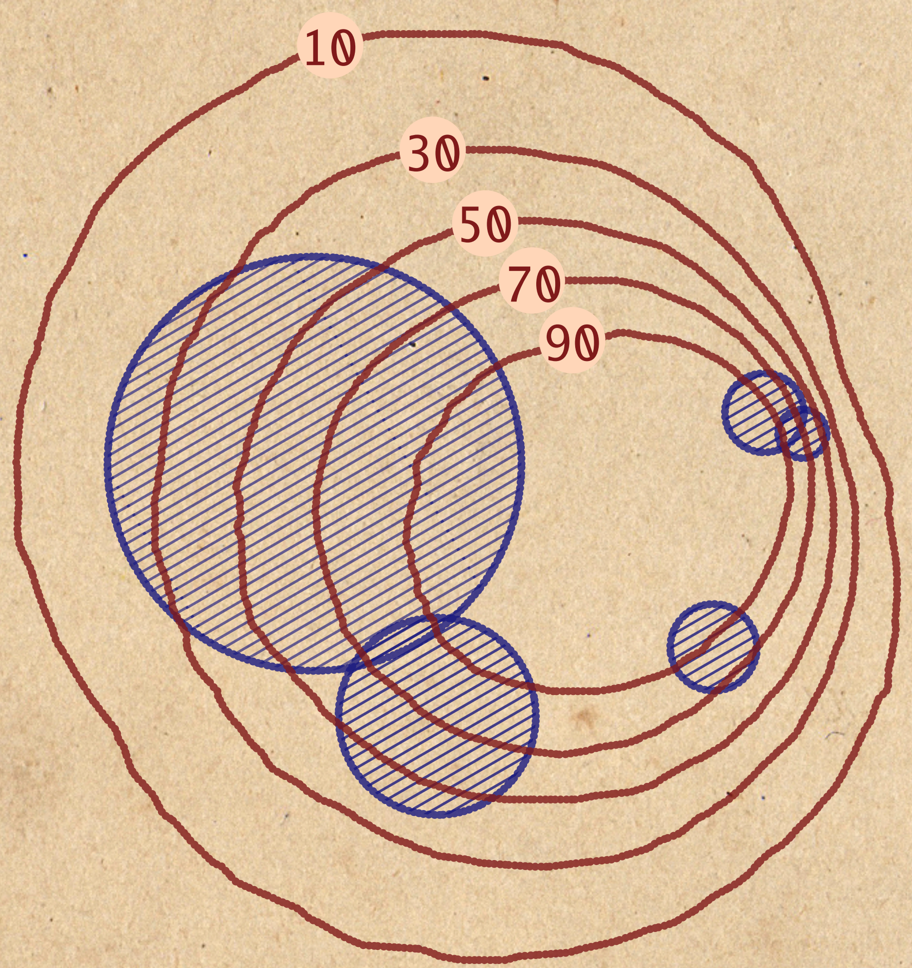
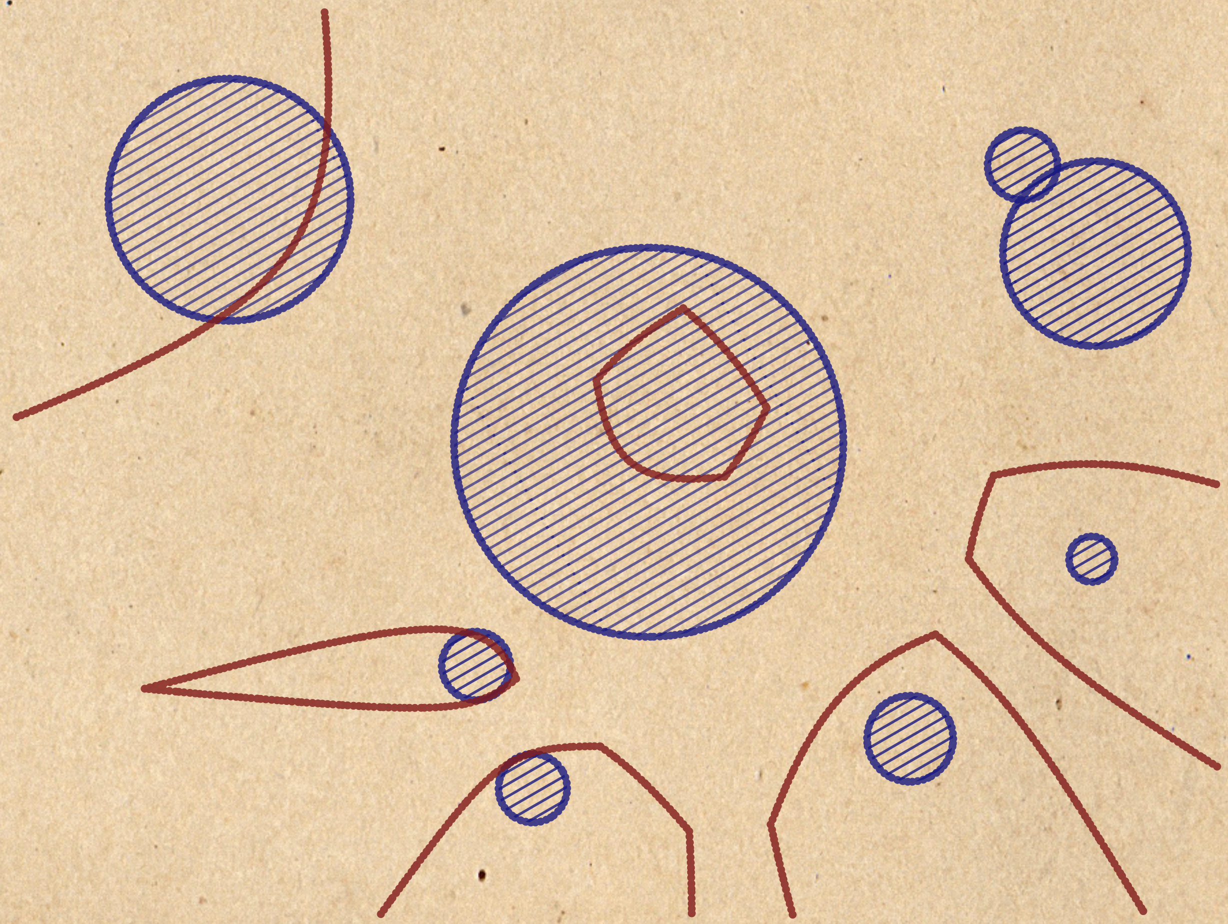
These are all very shapely things to show.





What if I’m interested in combinatorial information?

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Like, some sort of connectivity thing?

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Can we also visualise this “combinatorial uncertainty”?

DRAWING GEOMETRIC UNCERTAINTY GRAPHS

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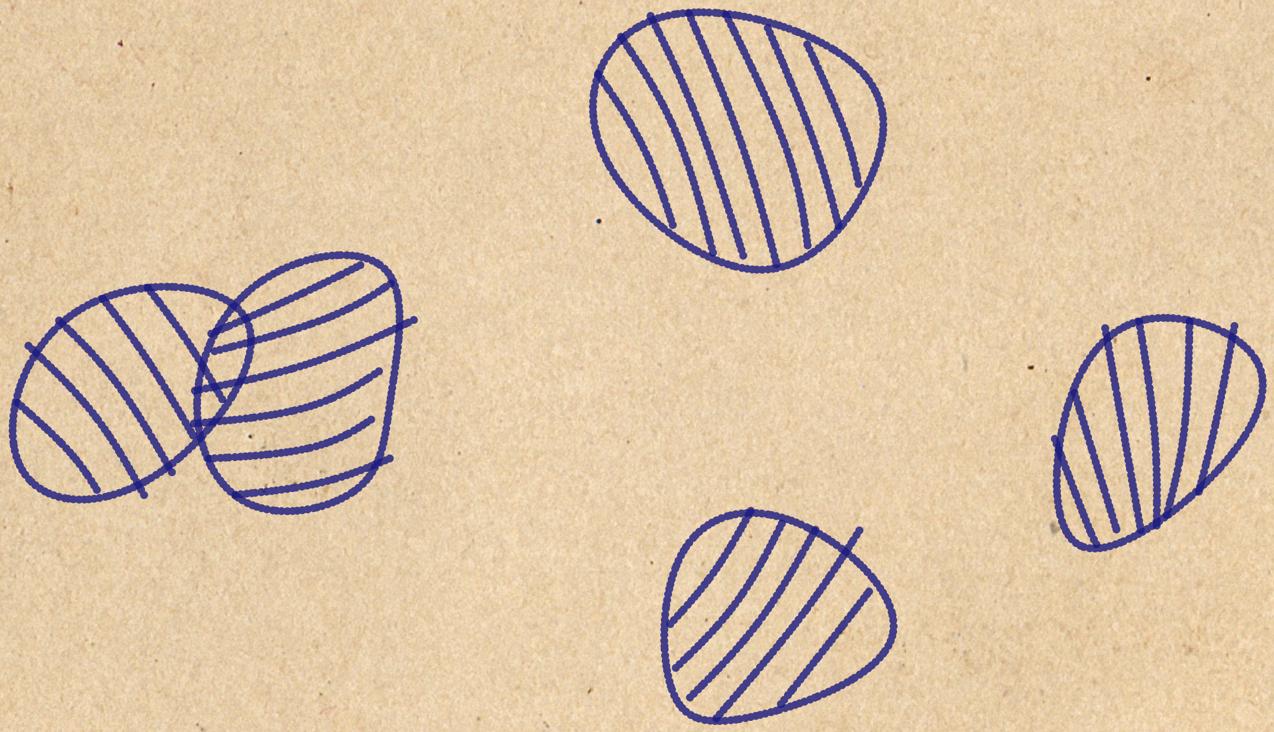

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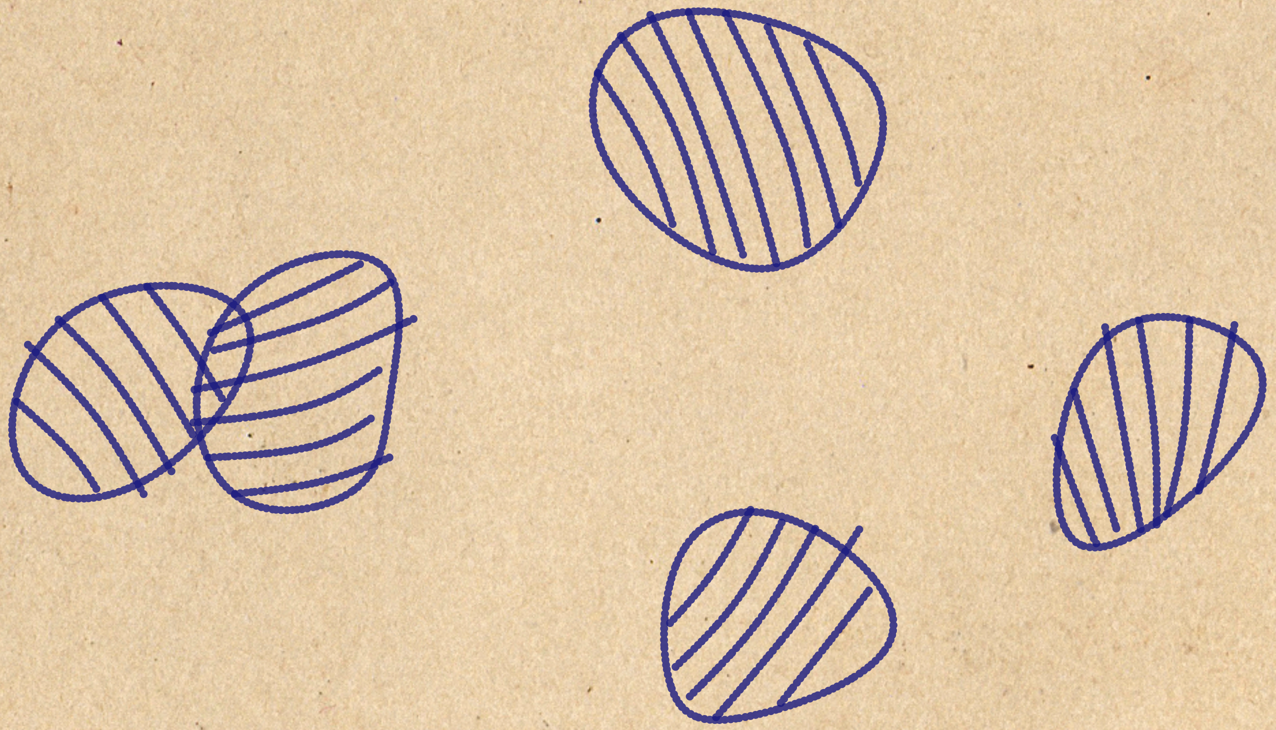
We can try a few different possibilities.



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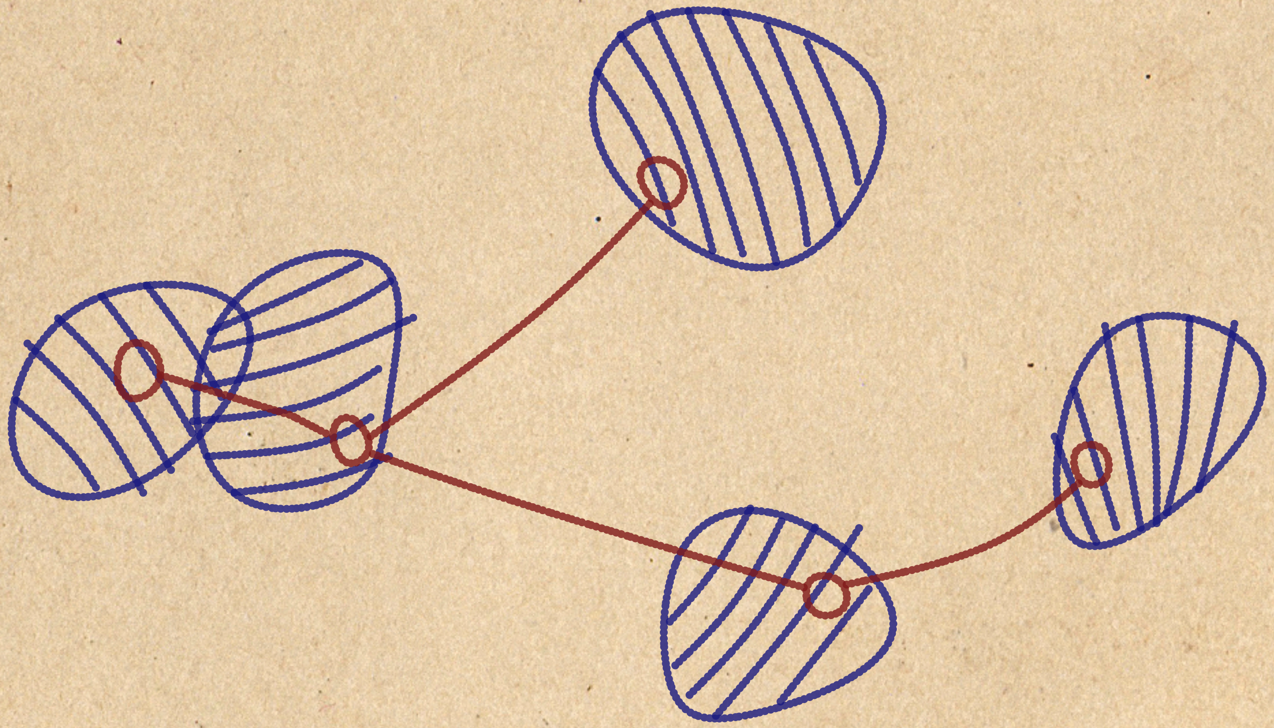

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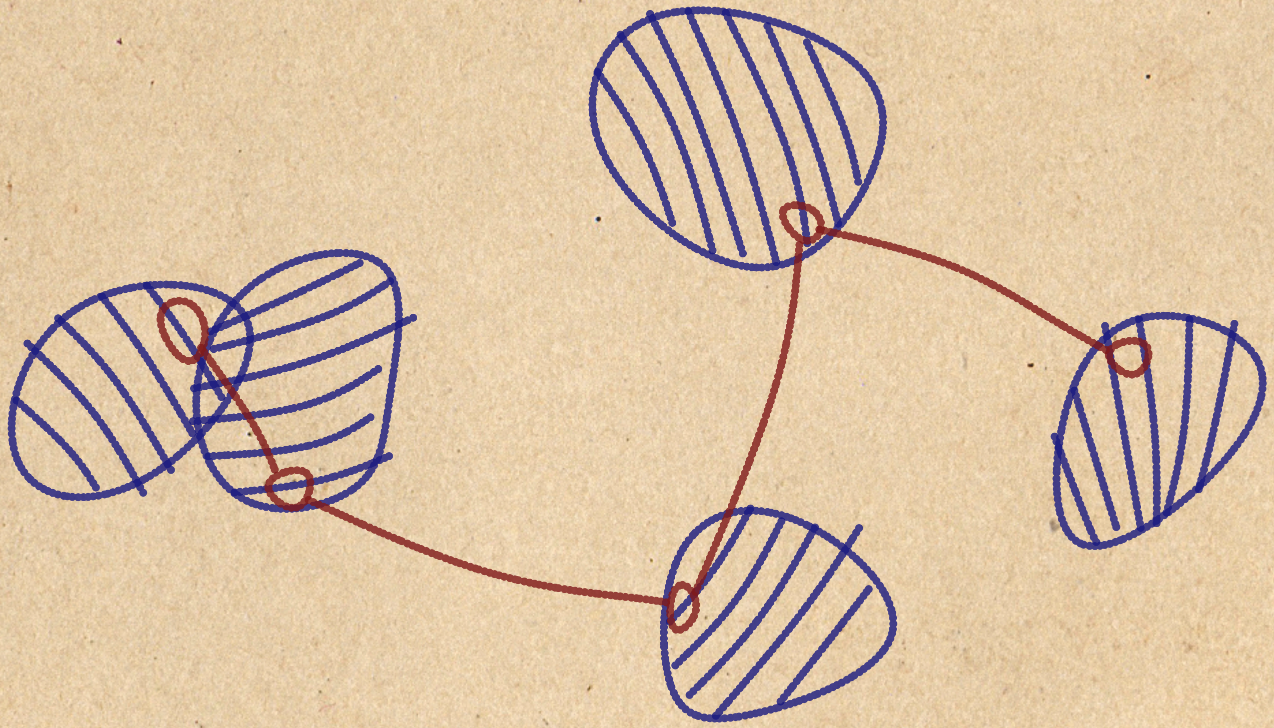

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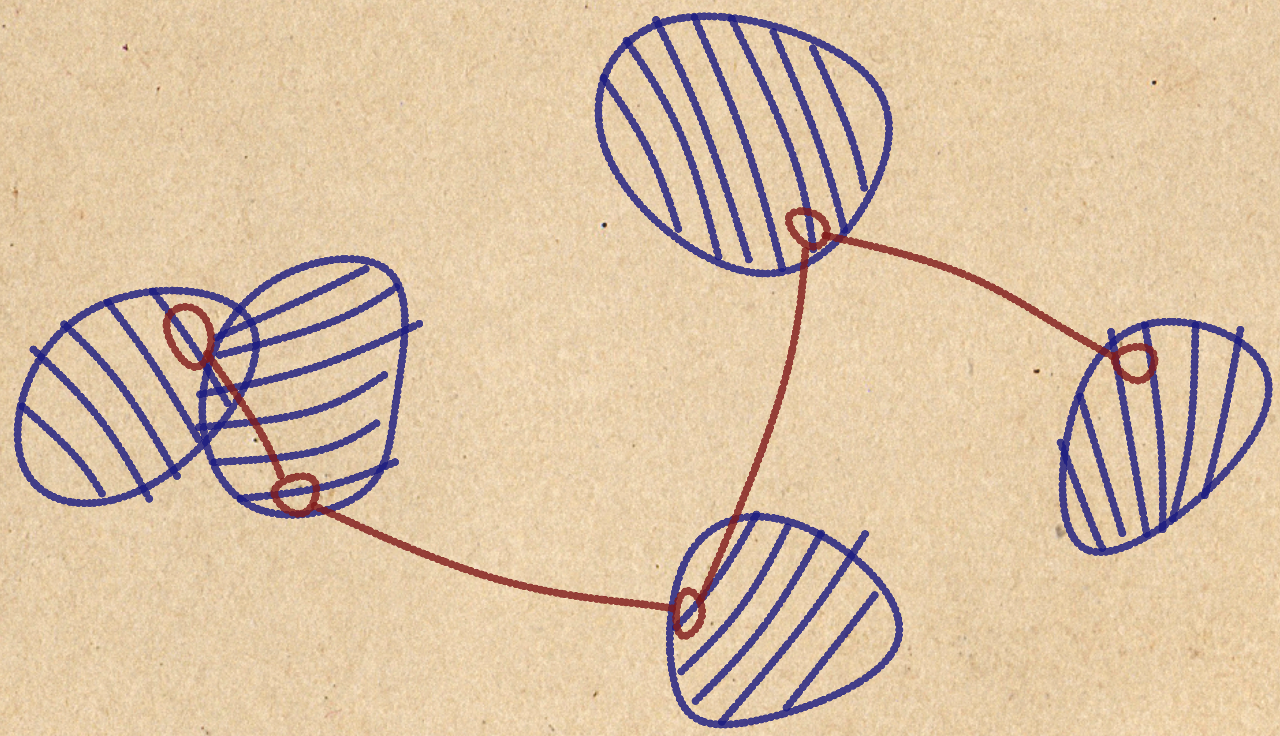
We can try a few different possibilities.

Or look at the graph of all *potential* edges.



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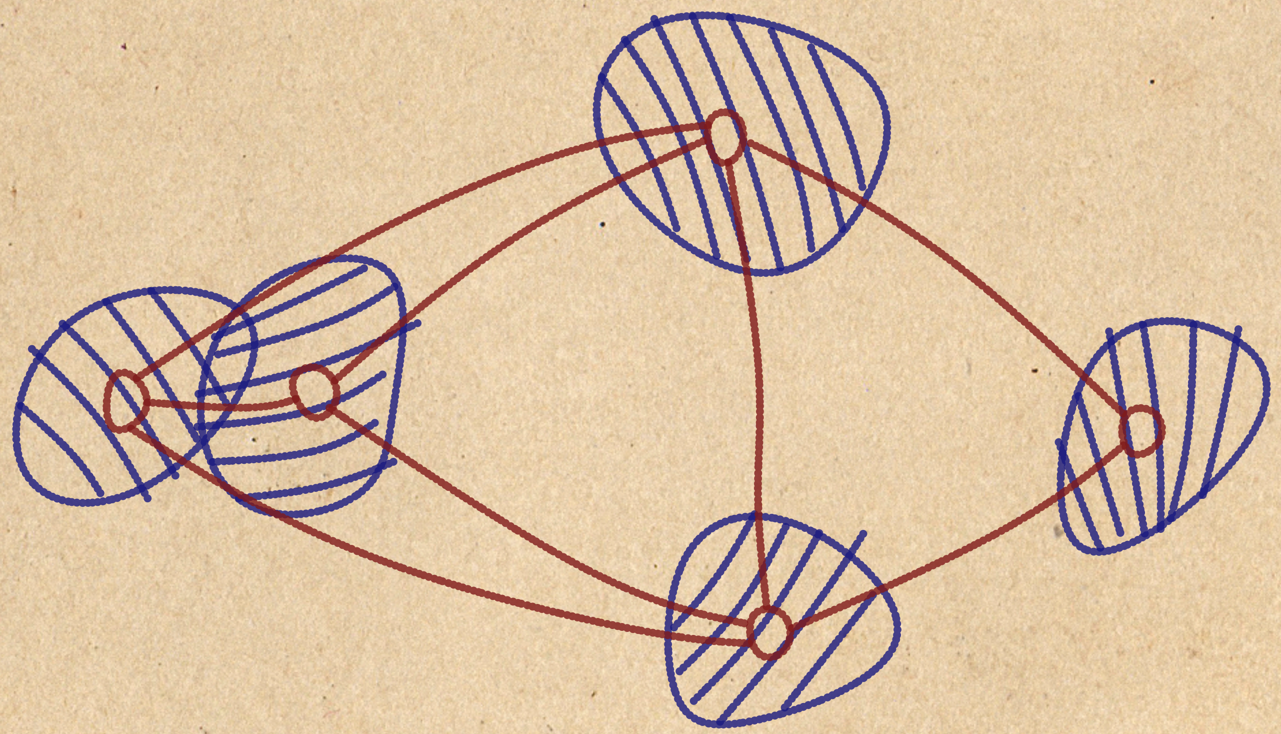

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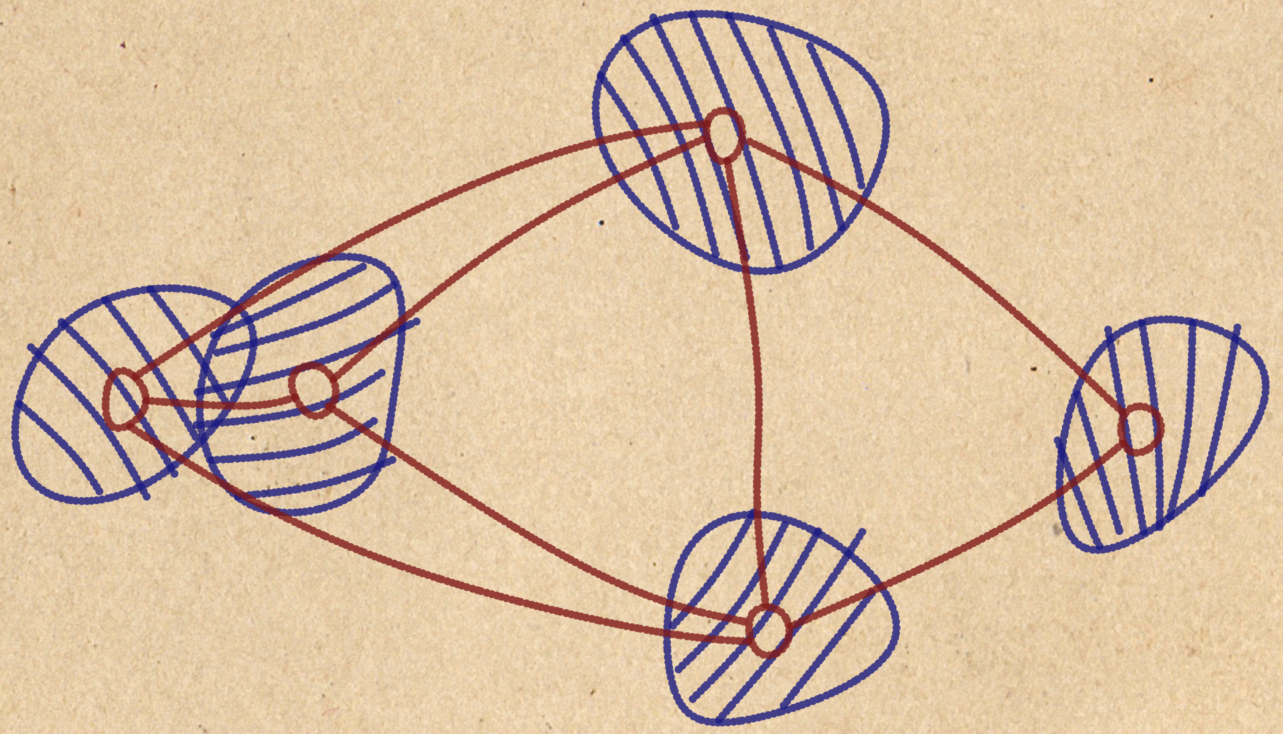

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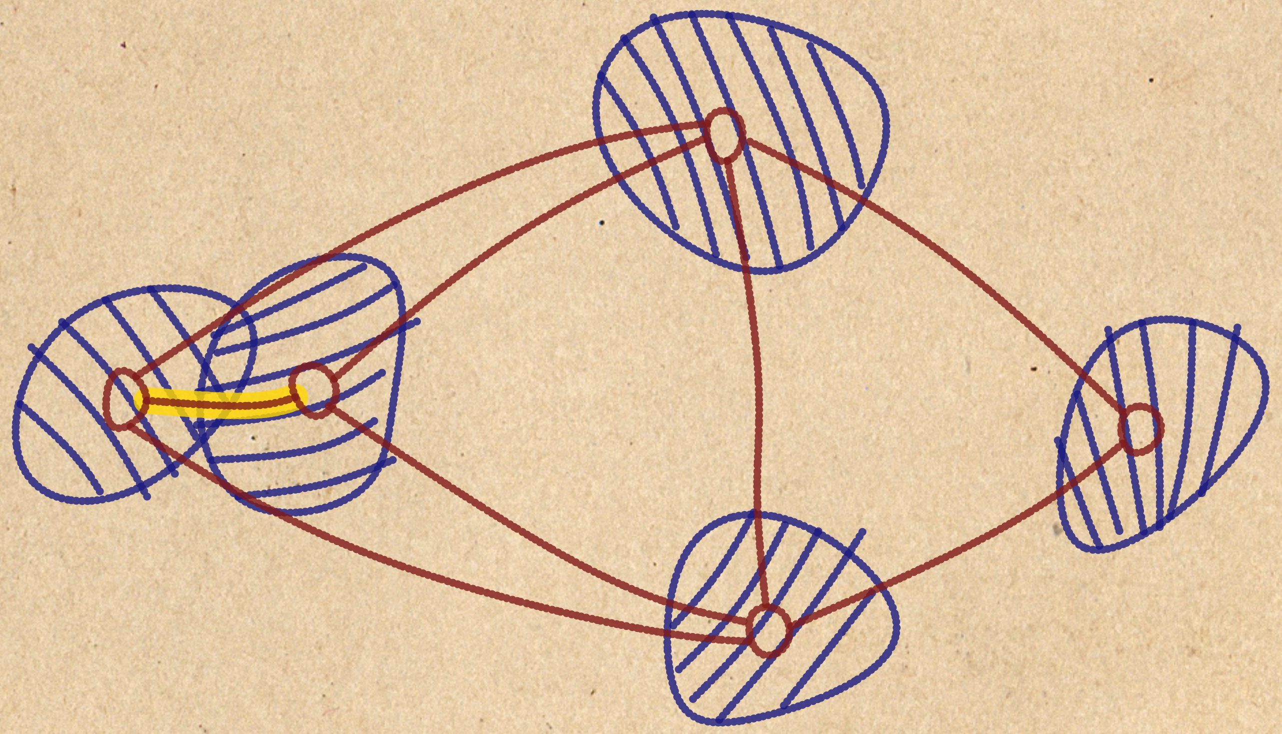

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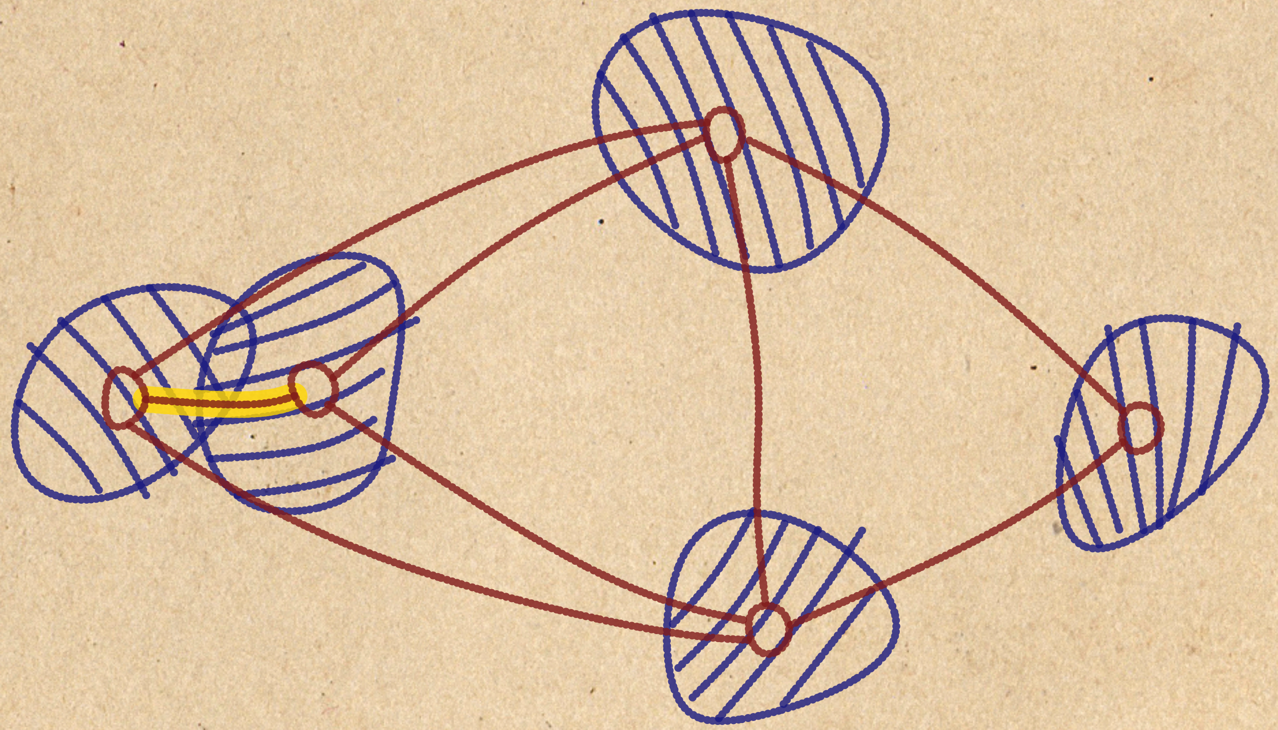

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But how do we draw this graph?

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VITAL QUESTION Does the geometric structure make these graphs easier to draw?

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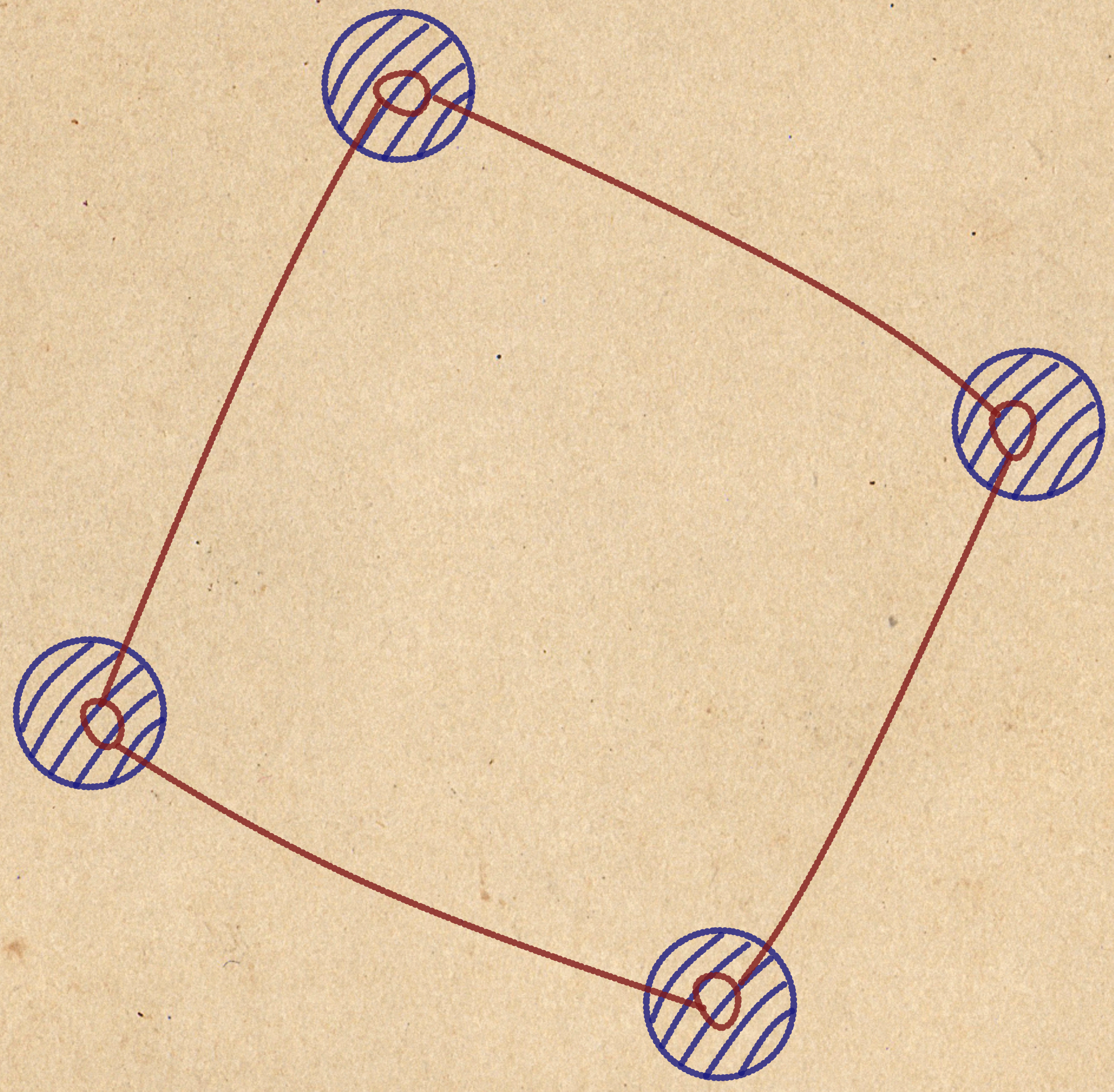

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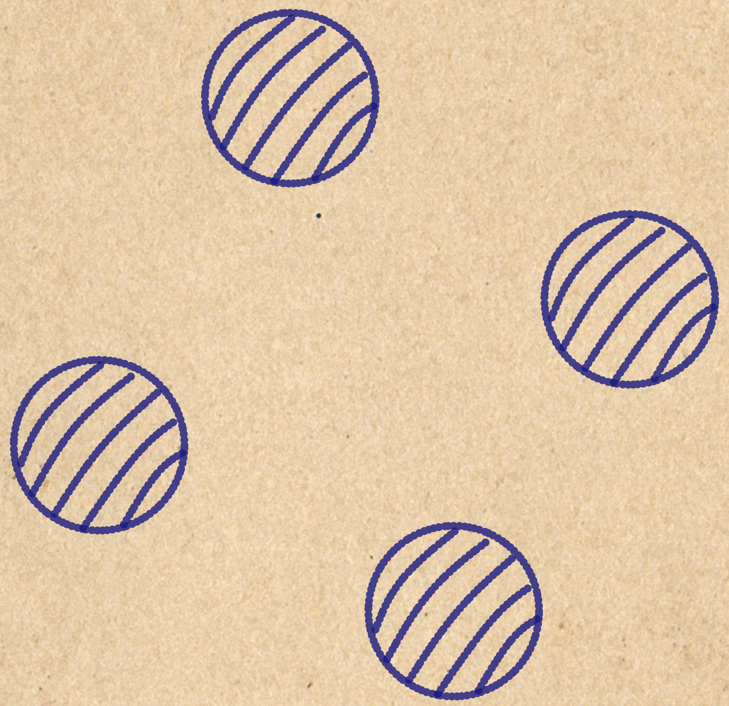

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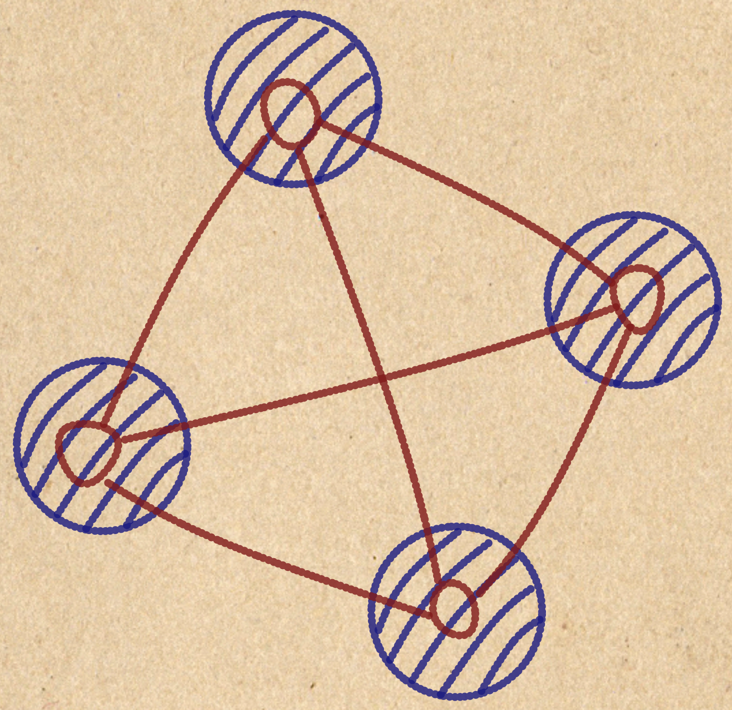

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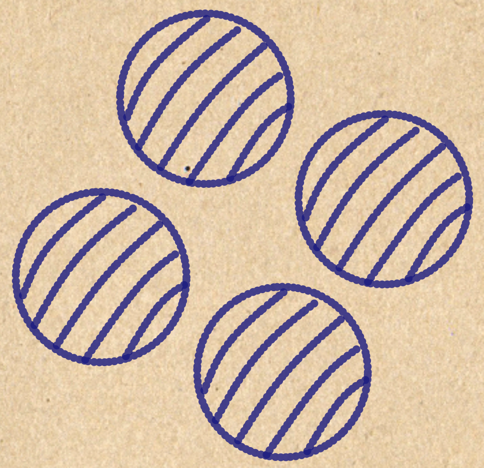

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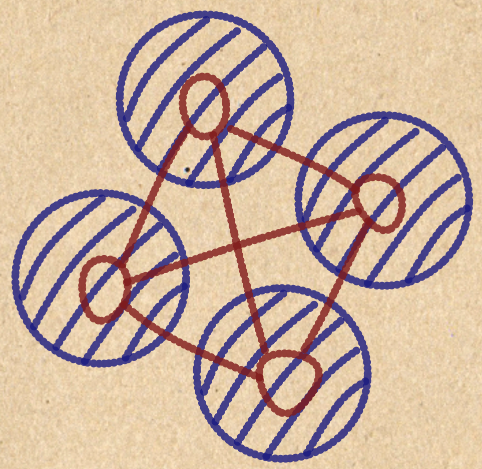

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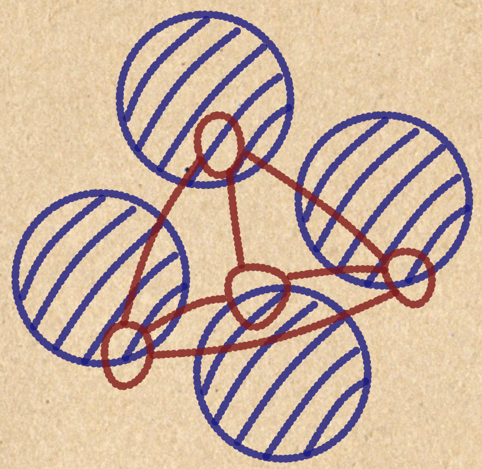

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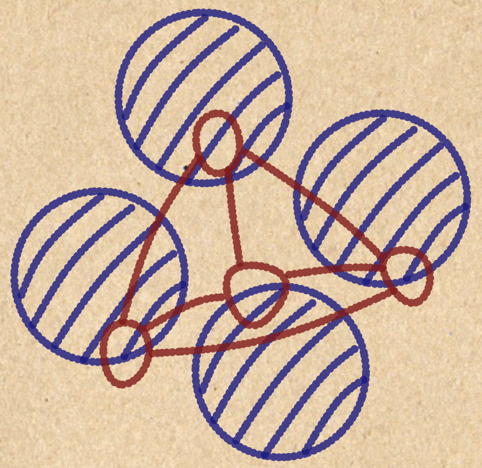

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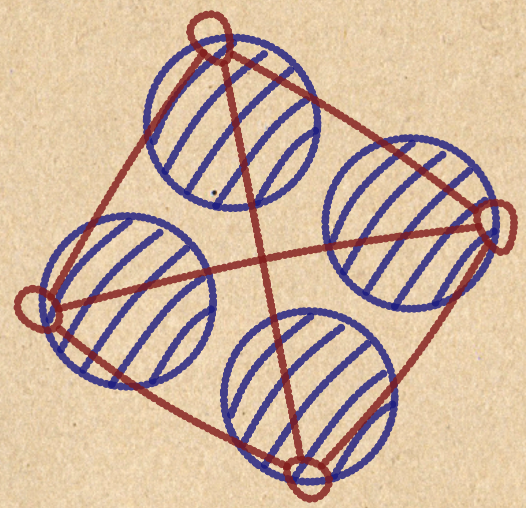

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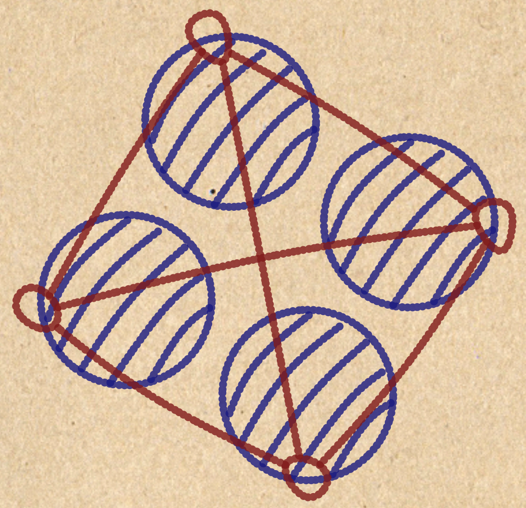

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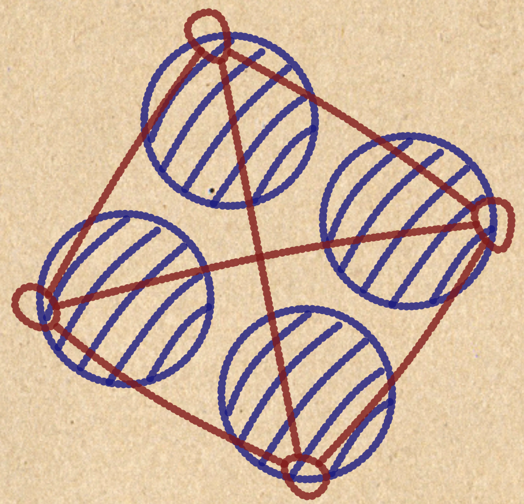

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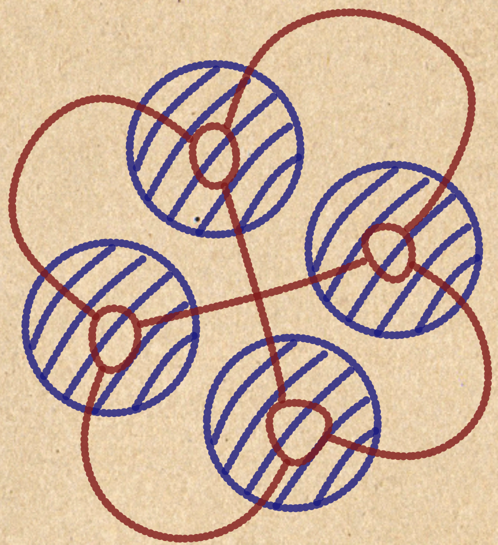

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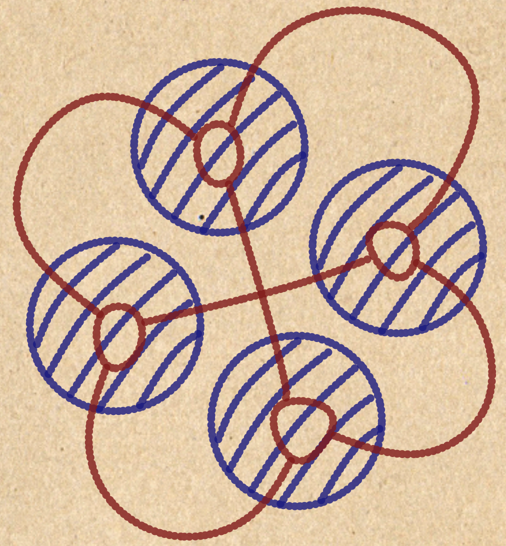

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OPEN PROBLEM Count the number of open problems this leads to.

Or consider the *convex hull*.

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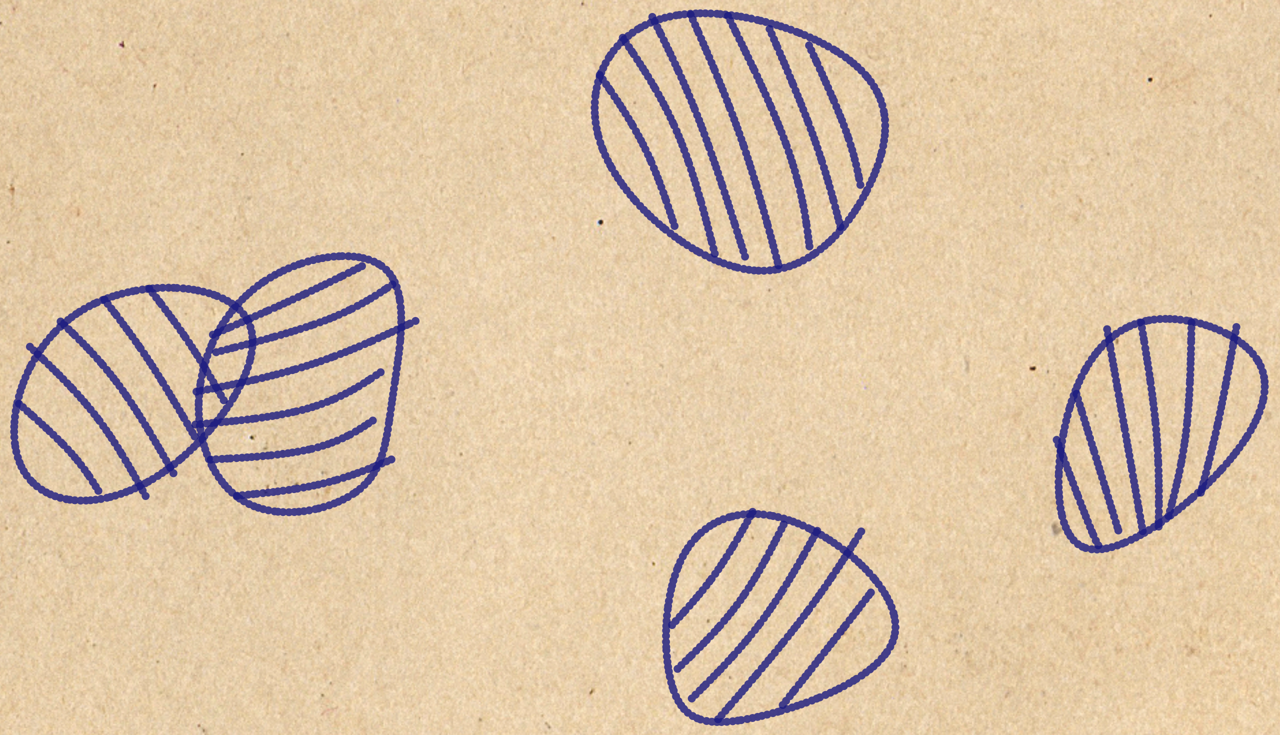

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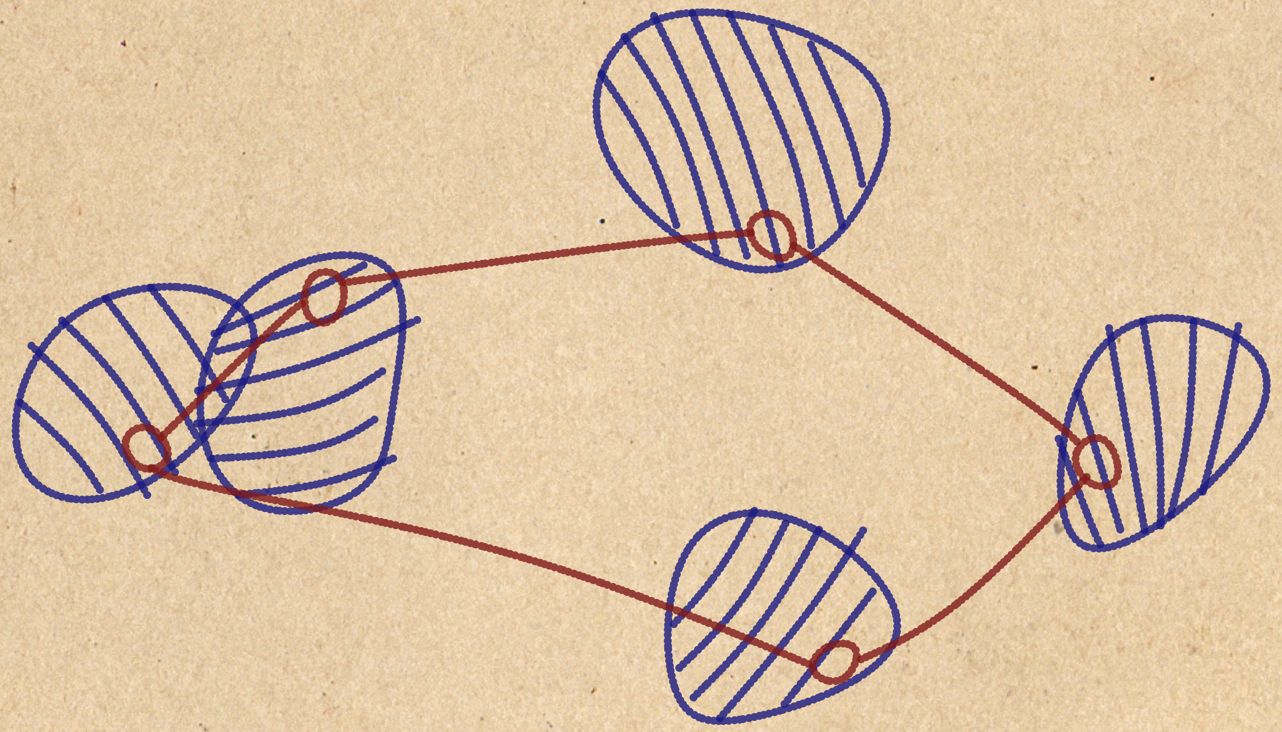

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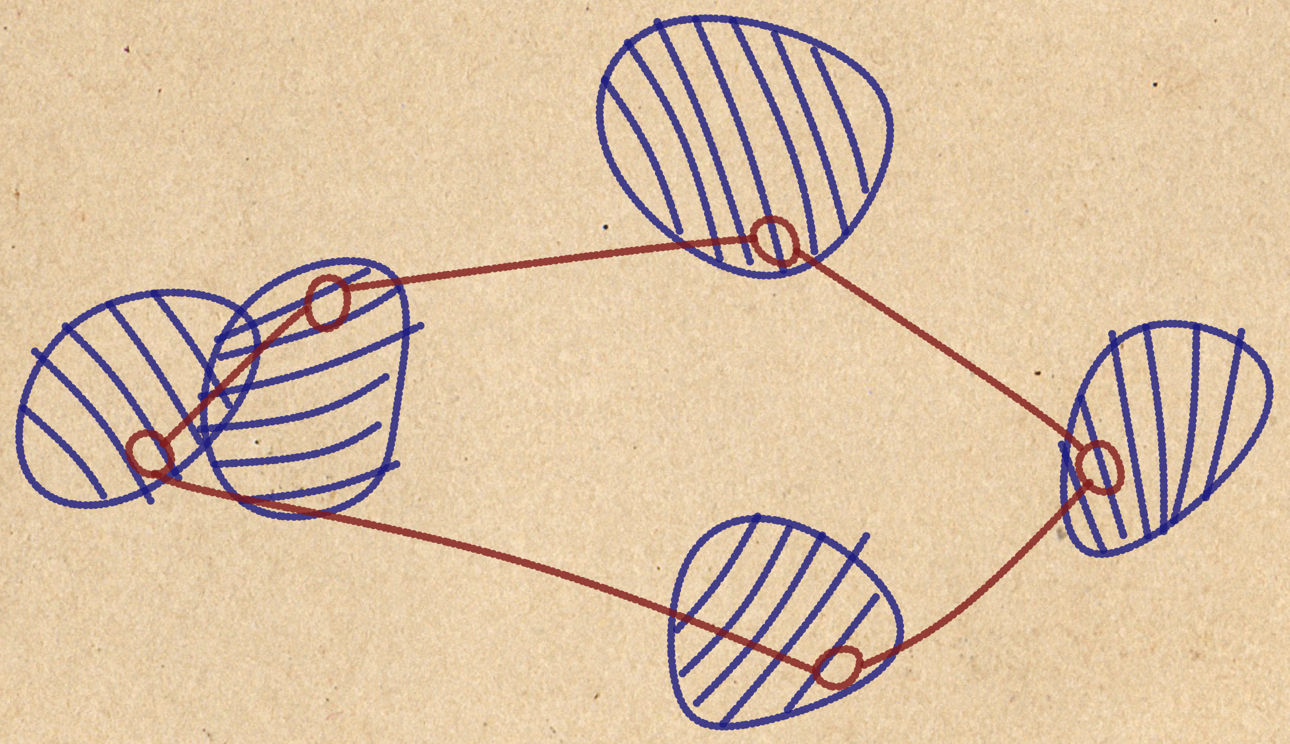
Again, there are different possibilities.



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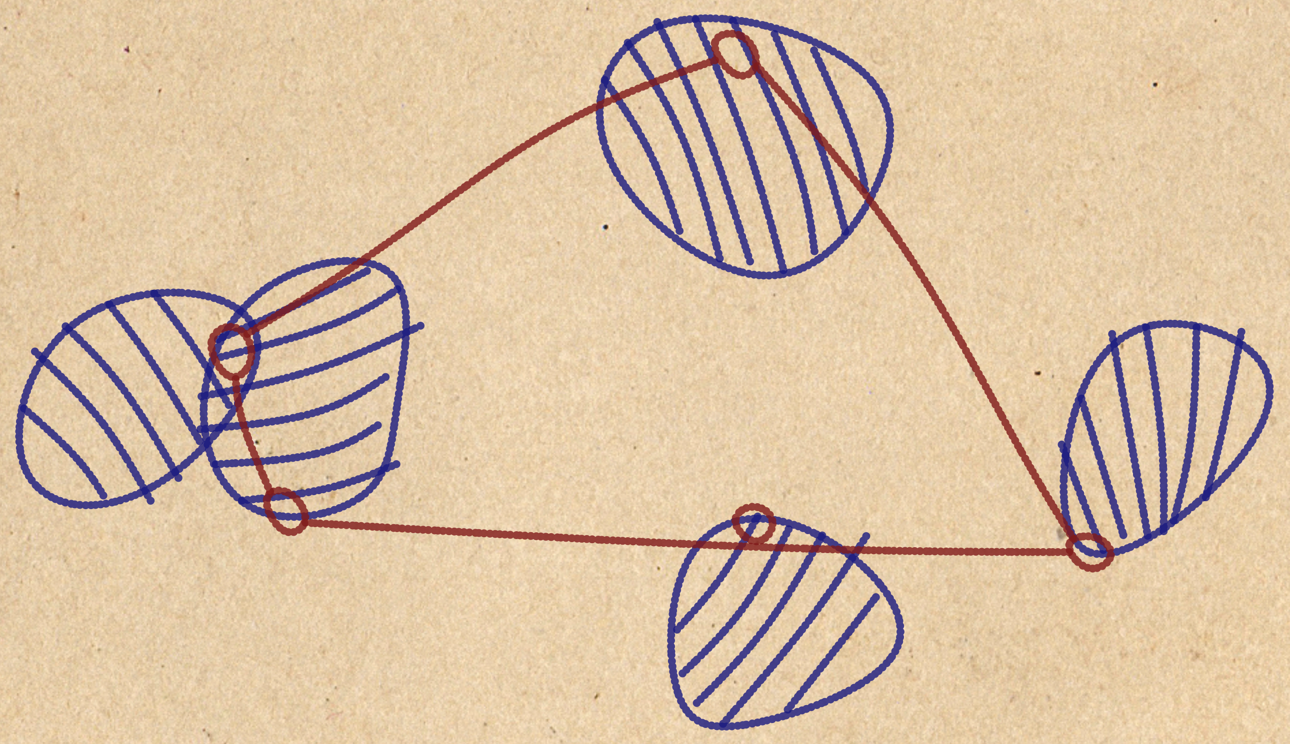

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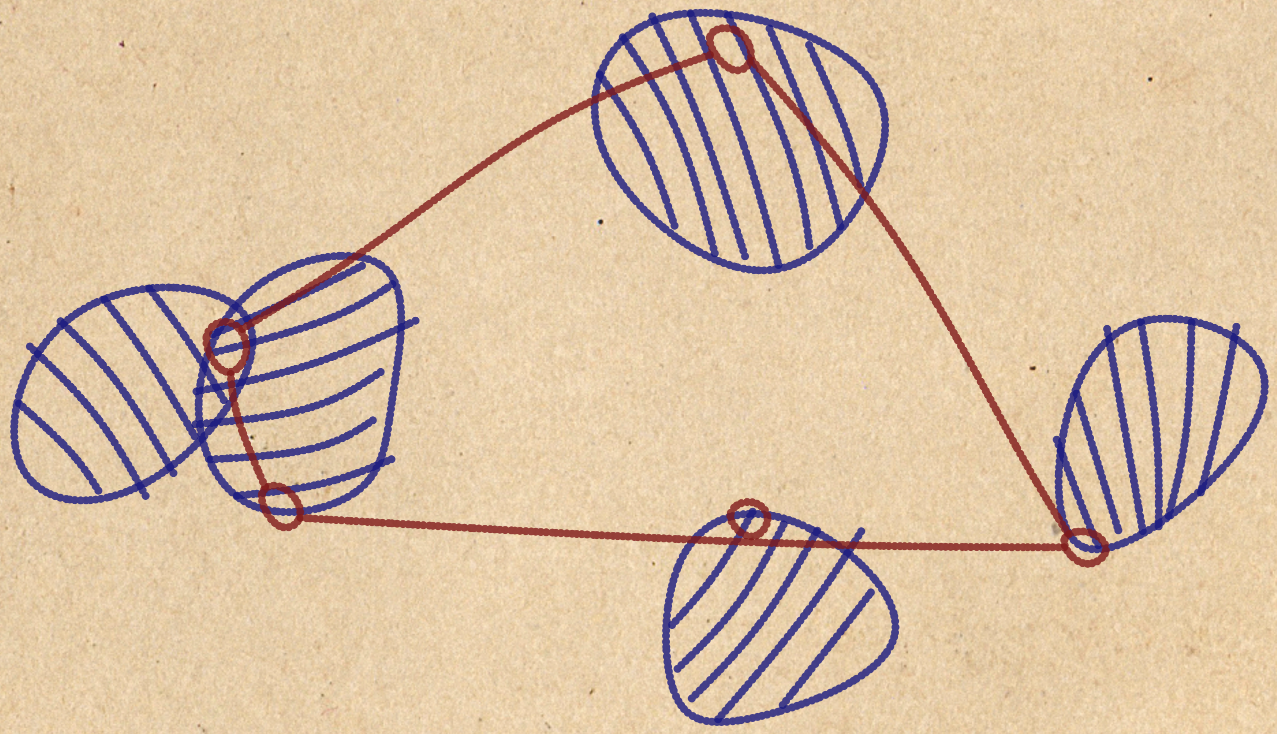
Again, we can draw the *potential* hull edges...



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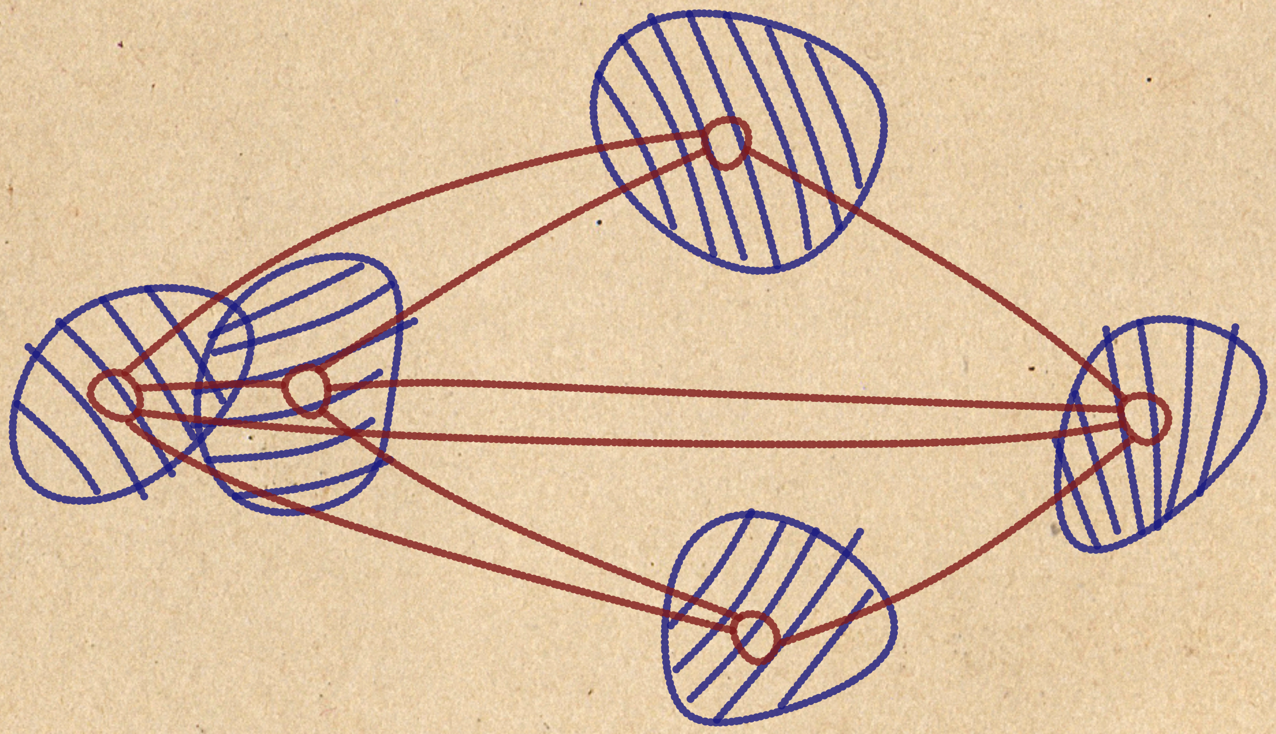

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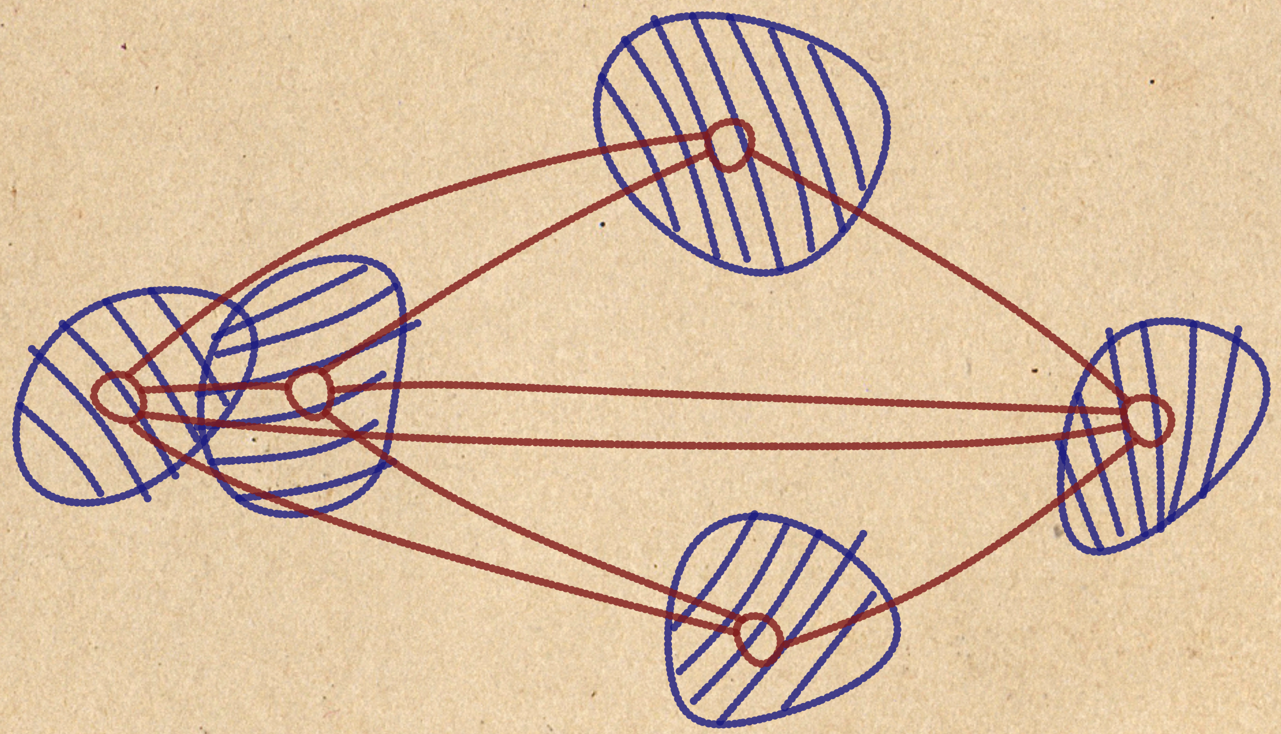
Again, we can draw the *potential* hull edges...

...and highlight the *guaranteed* hull edges.



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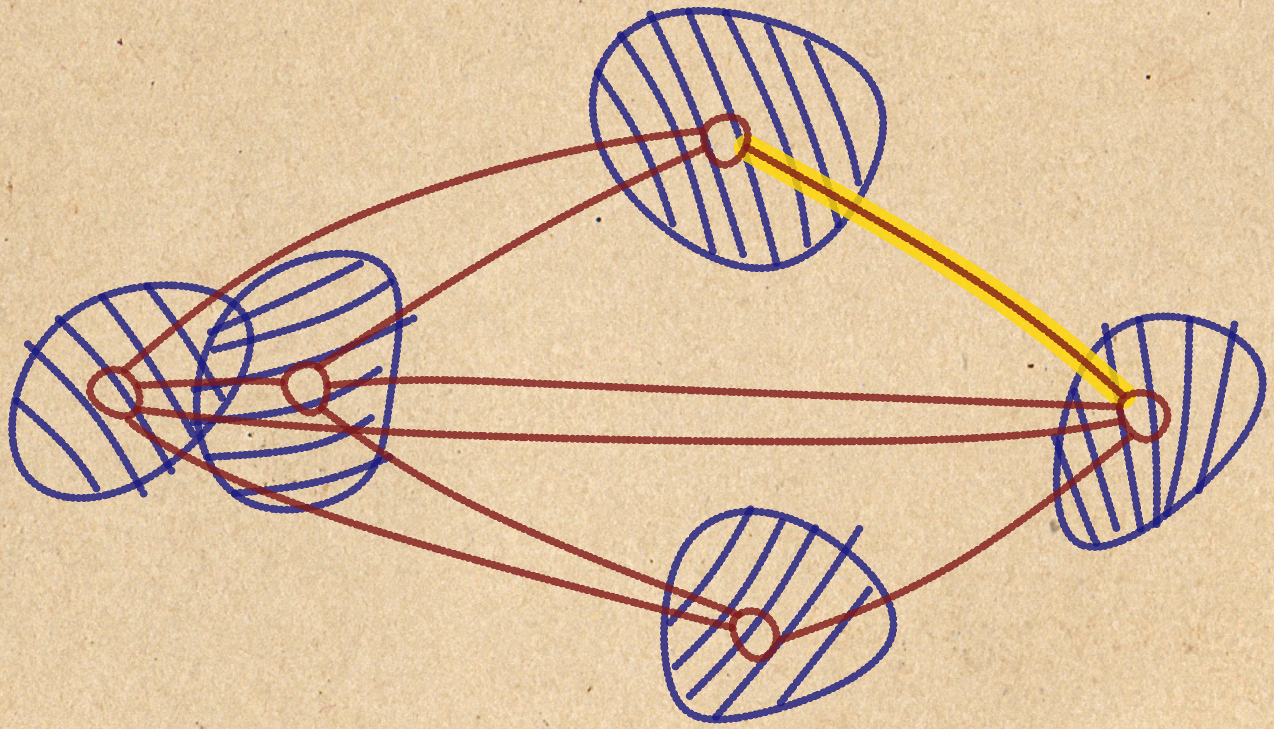

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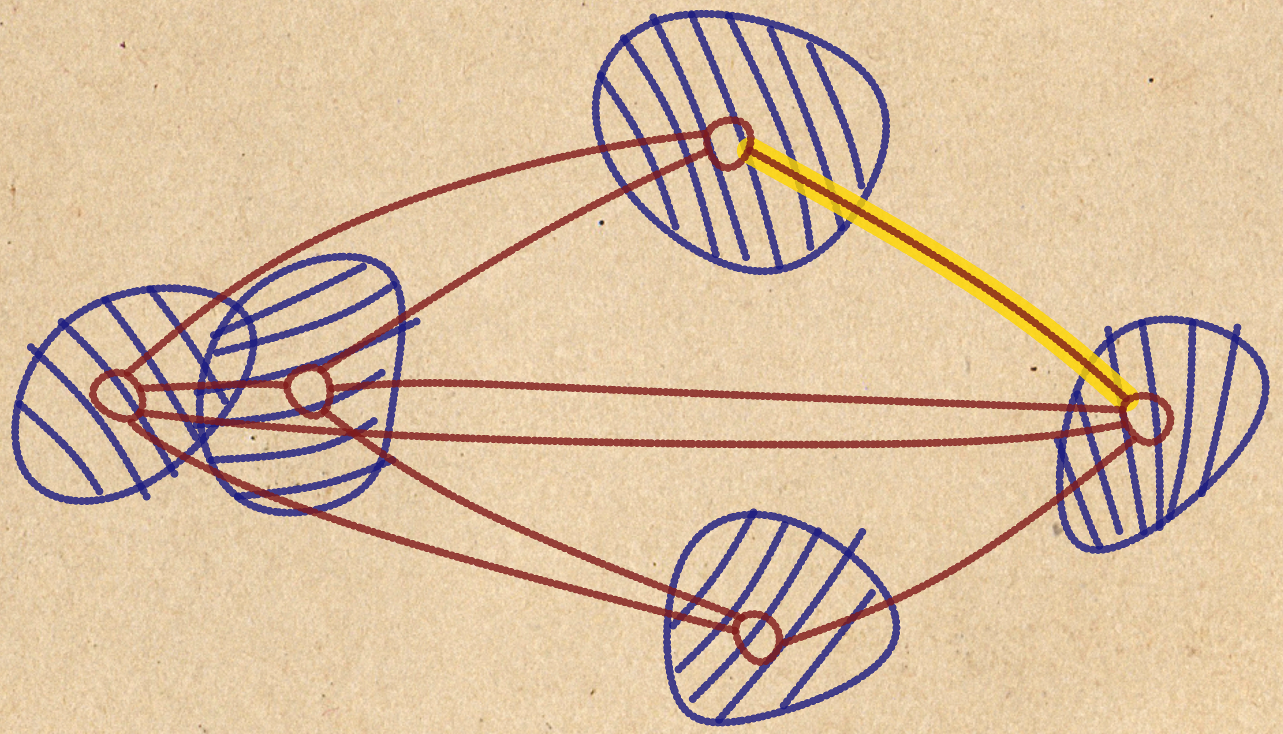

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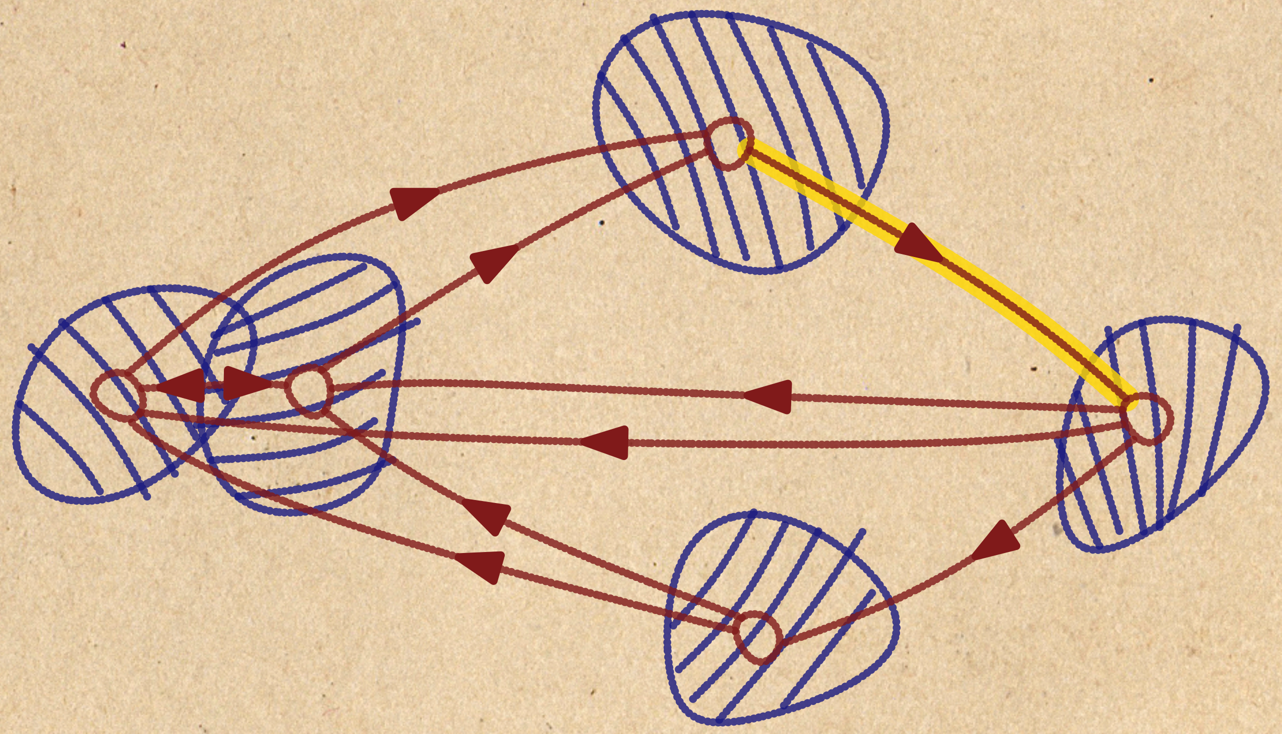

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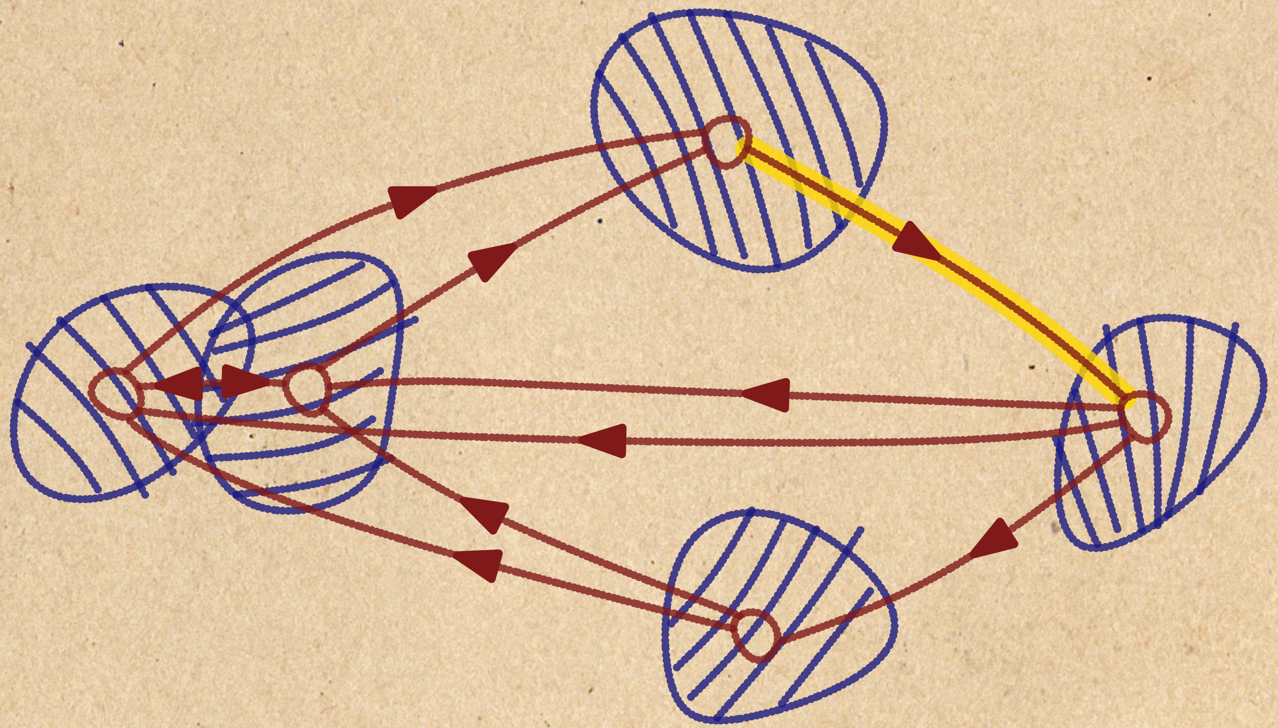

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OPEN PROBLEM Draw this graph nicely (and figure out what that means).

Or what about *unit disk graphs*.

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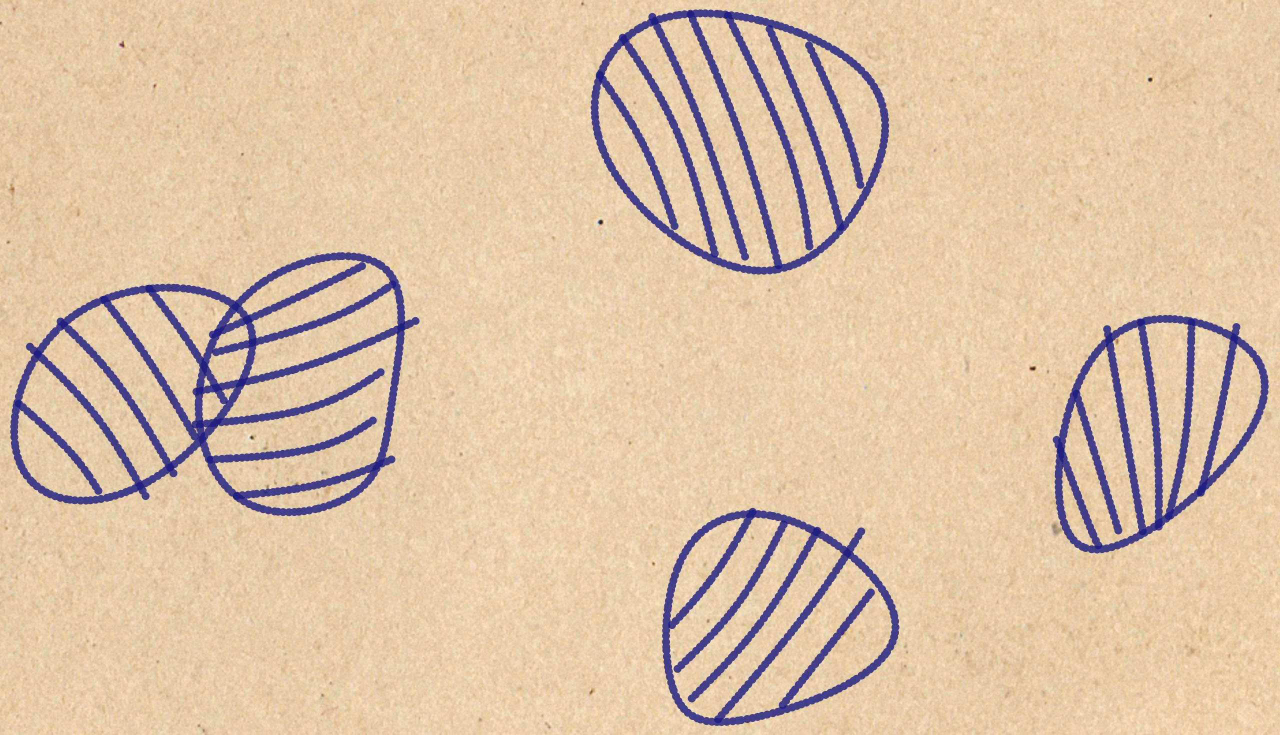

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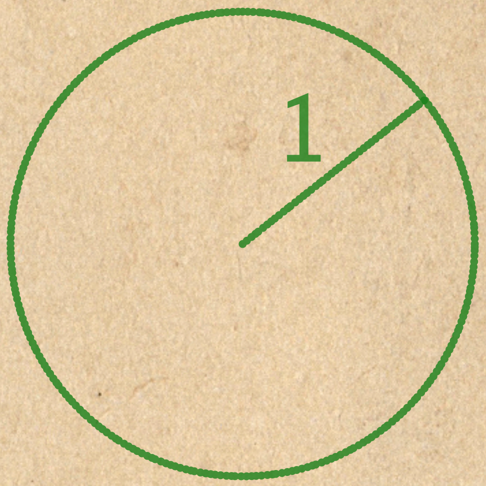

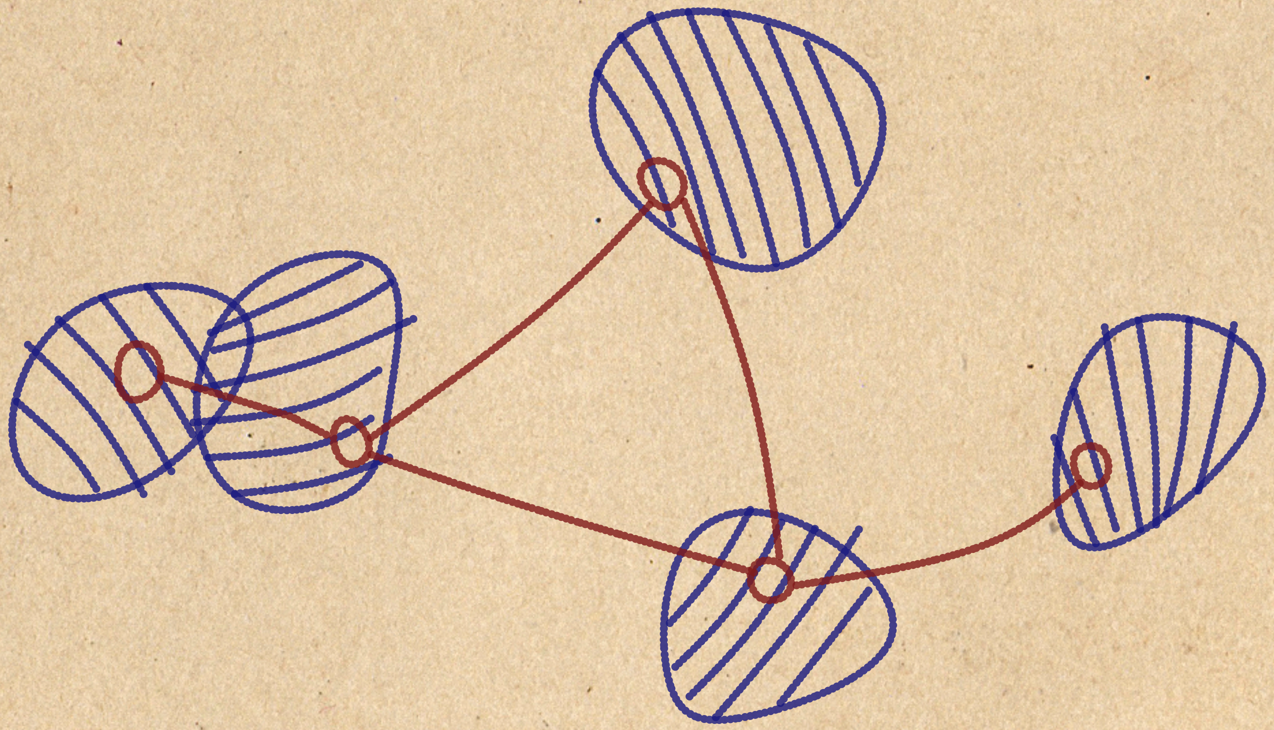

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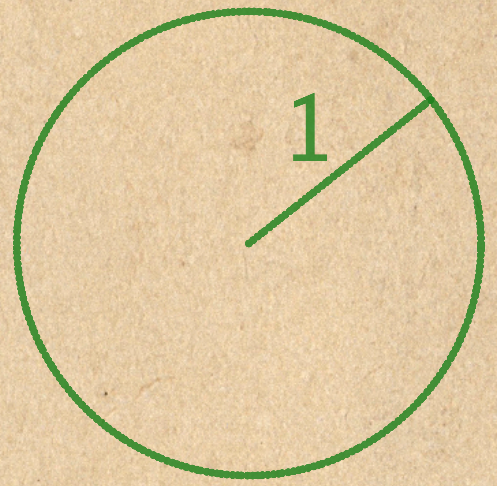

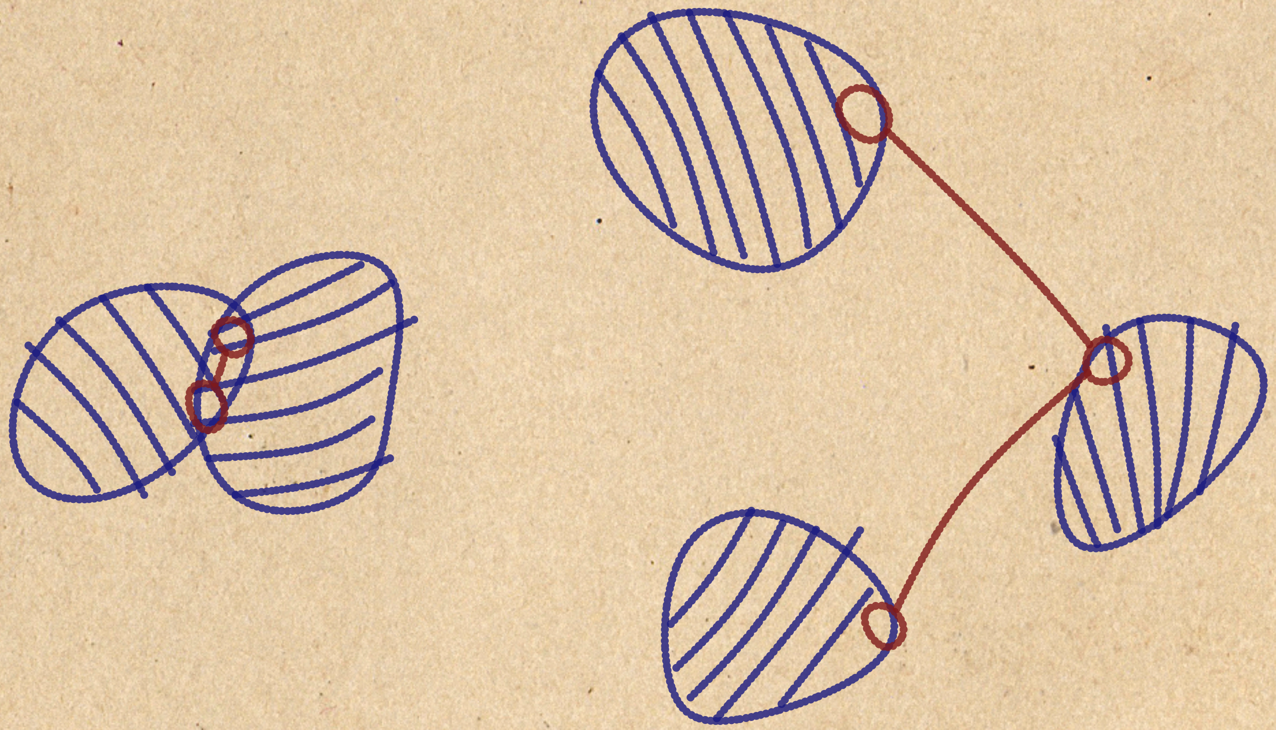

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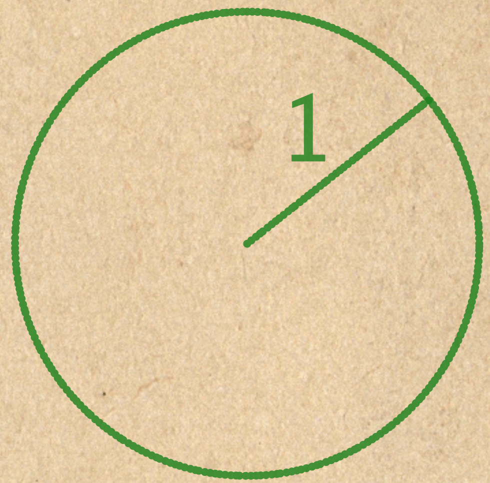
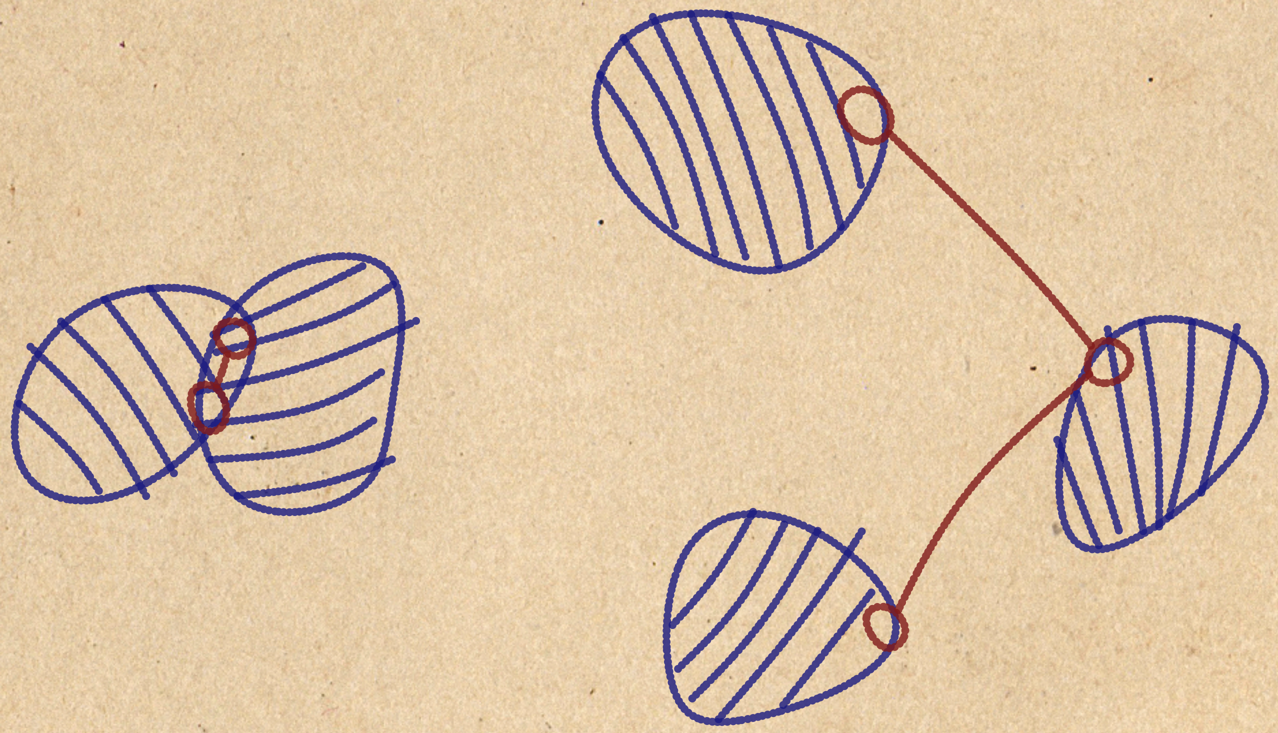
We can do the same thing!





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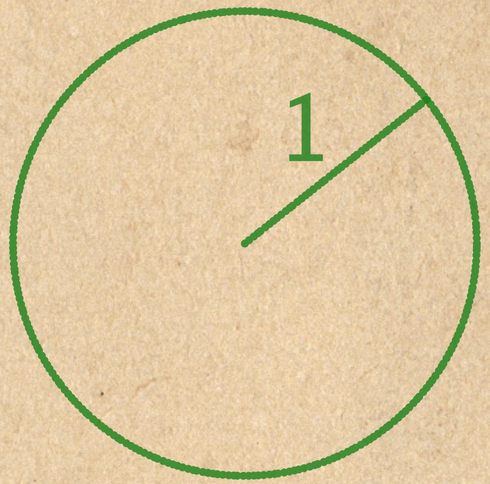

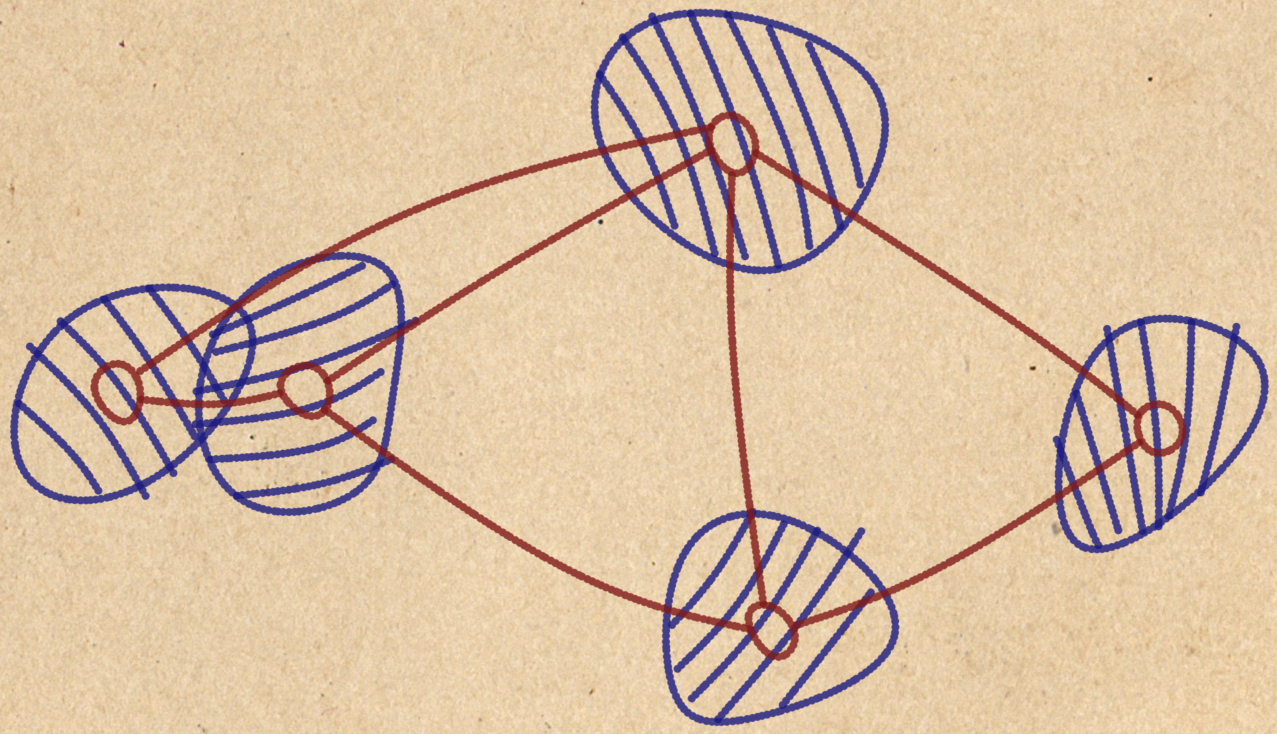

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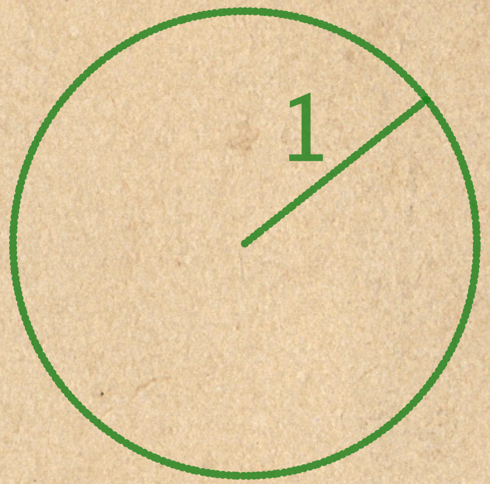

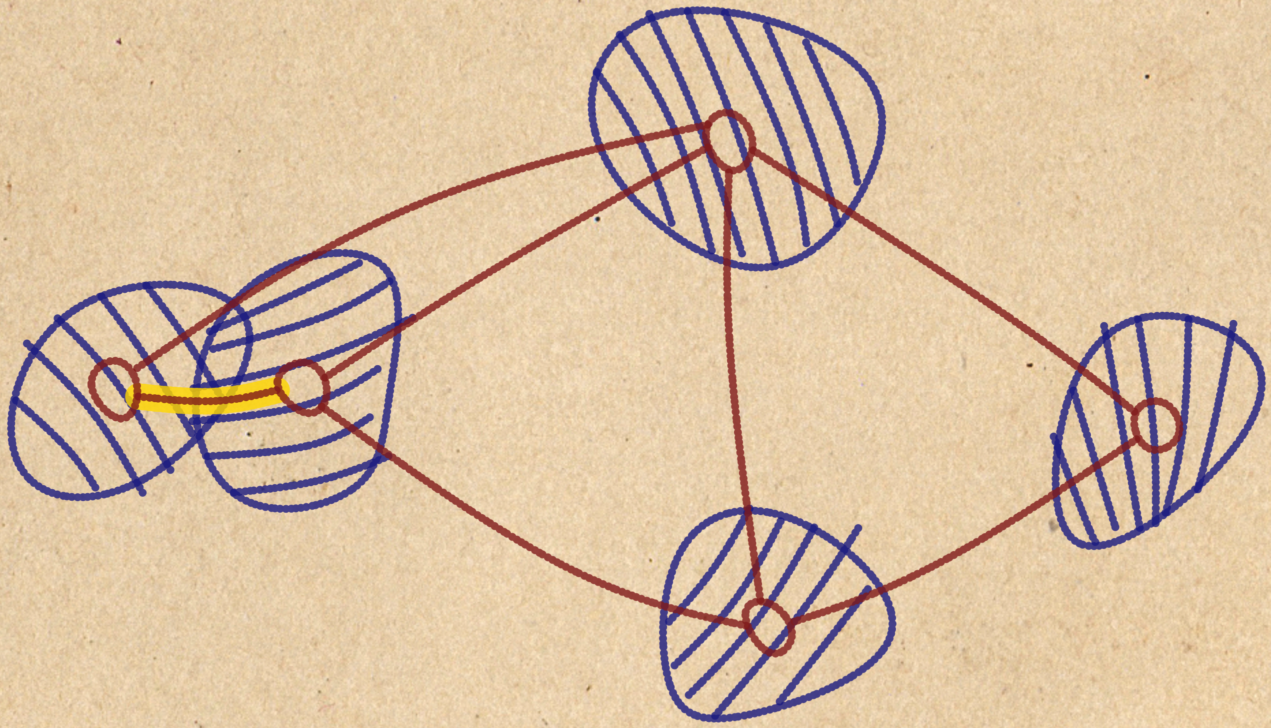

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OPEN PROBLEM What about general unit disks?

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OPEN PROBLEM What about general unit disks?

OPEN PROBLEM What about 1 > 0?

What is *your* favourite geometric graph?

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Delaunay triangulation?

What is *your* favourite geometric graph?

Delaunay triangulation?

Gabriel graph?

What is *your* favourite geometric graph?

Delaunay triangulation?

Gabriel graph?

Travelling Salesman Tour?

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OPEN PROBLEM Draw the uncertainty version of your favourite geometric graph, using your favourite optimisation criterion!

I would like to take this opportunity to briefly reiterate my main points.


THANK YOU