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TIGHT LOWER BOUND FOR THE STATE COMPLEXITY OF SHUFFLE OF REGULAR LANGUAGES

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ABSTRACT

The upper bound for the state complexity of the shuffle of two regular languages is $2^{mn} - 1$. We prove that this bound can be reached for some (not necessarily complete) deterministic finite automata with, respectively, m and n states. Our construction uses an alphabet of size 5.

Keywords: Finite automata, shuffle, state complexity

1. Introduction

By the state complexity of an operation on regular languages we mean a function that associates to the sizes of the DFAs representing the operands of the operation the minimal number of states of a DFA representing the resulting language in the worst case. Tight lower bounds for the state complexity of many basic operations on finite and infinite regular languages are obtained in [1, 3, 4] and in the references listed there.

Here we consider the operation of shuffle $\sqcup\!\sqcup$. If L_1 and L_2 are accepted, respectively, by an m-state DFA and an n-state DFA it is easy to see that $L_1 \sqcup\!\sqcup L_2$ is always accepted by a DFA having at most $2^{m \cdot n} - 1$ states. We consider DFAs that are not necessarily complete. We show that this upper bound is tight, that is, we construct an example for which the lower bound for the number of states of a DFA accepting $L_1 \sqcup\!\sqcup L_2$ reaches the value $2^{m \cdot n} - 1$.

The DFAs used in our construction are incomplete (that is, we allow undefined transitions). The construction gives a lower bound also for the shuffle of a complete m-state DFA and a complete n-state DFA but it does not reach the corresponding